Max Born, Göttingen and Quantum Mechanics

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1 Introduction

We are here to celebrate the hundreth anniversary of the official start of the *Institute for Theoretical Physics* at the University of Göttingen. In order to set the stage I'm going to look back to the beginning of the twentieth century and will mention a few things related to the title of my talk that happened in the year 1900:

Max Born (1882-1970) was close to finishing school in his hometown Breslau and planned to study mathematics there. The University of Göttingen was by far the best place in Germany to do so, but Max Born did not know that yet [1].

One of Göttingen's math professors David Hilbert (1862-1943) presented ten unsolved problems (twenty-three in the published version) at the Second International Congress of Mathematicians in Paris in August 1900, later called Hilbert's problems. Hilbert's name is best known to physics students from the *Hilbert space* concept used in quantum mechanics.

Theoretical physics in Göttingen was represented by Woldemar Voigt (1850-1919). Because of my restricted time I cannot discuss his achievements here.

In October 1900 Max Planck (1858-1947) presented his formula for the black body radiation. This is considered the beginning of quantum theory [2].

Max Born will be center stage in my talk. I will begin with his early career before the Göttingen Institute of Theoretical Physics was established by him.

2 Max Born: His early career

After having started studying mathematics in Breslau Born spent two summers at the University of Heidelberg and the ETH Zurich. In Heidelberg he met his later friend James Franck for the first time. After returning to Breslau his companion Otto Toeplitz advised Born to go to Göttingen if he wanted to hear lectures of the same quality as at the ETH Zurich. Born did not even know where Göttingen was located. When he arrived there in 1904 the mathematics faculty consisted of four great figures: Felix Klein, David Hilbert, Herrman Minkowski and Carl

Runge (arranged according to their “call” to Göttingen) called “die Bonzen” (big shots) by the students.
Born attended lectures by David Hilbert. Hilbert liked the notes Born took and asked him to post them in the reading room of the Mathematics Institute. This led to an early close contact to Hilbert. Born’s step mother had met Herrmann Minkowski in Königsberg when they were young. For this reason Born learned to know Minkowski better than the usual student. For the PhD thesis Hilbert proposed Born to find out if the zeros of the Bessel functions are transcendental numbers. As Born made no progress he had doubts about his future as a mathematician.
The relation to Felix Klein was not without problems. The faculty proposed the topic “The stability of the elastic line” for the “Academic Prize 1906” and Klein suggested to Born to compete. Born responded that he had no deep interest in elastic problems and would not do so. Klein was furious. Born’s friends convinced him to change his mind. In June 1906 the prize winner was presented: Max Born. His work was also accepted as his PhD thesis.

A short side remark: Klein some years earlier had had a special assistant for his lectures: Arnold Sommerfeld (1868-1951) who plays an important role later in the talk. He came to Göttingen in 1893 and shortly later became assistant to Felix Klein with whom he wrote books “Über die Theorie des Kreisels” (theory of tops), the first of four volumes published in 1897. For his Habilitation Sommerfeld had submitted the “thesis”: Mathematical theory of diffraction”, the first rigorous solution of a diffraction problem. After some years on a professorship in Clausthal he accepted the chair for Technical Mechanics at the RWTH Aachen in 1900. He later moved to the chair for Theoretical Physics at the LMU Munich in 1906 where he stayed for the rest of his life, despite various interesting offers.

After having received his PhD Born returned to Breslau for his military service which had been postponed after finishing school because of his asthma problem. An asthma attack soon finished Born’s military service. His friend James Franck who had decided to become an experimental physicist advised to Born to visit Cambridge to learn more about physics. Back home Born started to work on problems related to special relativity. He knew about Minkowski’s four-dimensional space-time approach and contacted him because he had diffculties solving the problem he worked on. To Born’s great surprise Minkowski offered Born to come to Göttingen to work with him on the problem.

So Born’s second longer stay in Göttingen started in December 1908. But it had a sad beginning. Minkowski died from appendicitis on January 12, 1909.
Born stayed in Göttingen to obtain his Habilitation to become a lecturer. His inaugural lecture was on J.J. Thomson’s “plum pudding” atomic model in which electrons as point particles are assumed to be in a homogenous spherical positive

charge background. Thomson's assumption about the positive charge is strong contrast to Rutherford's model with the positive charge in a nucleus of much smaller size than the atom. After Rutherford's scattering experiments in 1912, confirming the existence of a nucleus, the plum pudding model lost attention. In solid state physics a homogenous positive charge background is introduced in the *jellium model*, sometimes used to simplify the theoretical description of metals.

In the years 1912-1913 Born collaborated with Theodore von Kármán.

In 1913 Max Born married Hedwig Ehrenberg the daughter of a law professor in Leipzig.

The first of the many books Born wrote appeared in 1915: "Dynamik der Kristallgitter (The Dynamics of Crystal lattices)".

From 1915 to 1919 Born was associate professor ("Extraordinarius") in Berlin, where he met Max Planck and Albert Einstein. Einstein often came with his violin to the home of the Borns. He performed with Born playing the piano. They had a friendly relationship and exchanged letters until a few months before Einstein's death in 1955.

From 1919-21 Born was full professor ("Ordinarius") at the University of Frankfurt (Main). Max von Laue, a friend of Born, who held the Frankfurt chair before, wanted to work in Berlin. Born and Laue simply exchanged their positions, a rather unusual procedure.
Otto Stern was Born's research assistant in Frankfurt. Stern changed from theoretical physics to experimental physics during this time. He started experiments to study properties of atoms and molecules with the help of molecular rays, for which he received the 1943 Nobel Prize in Physics. Born was fascinated and as he had is own labaratory in Frankfurt he performed similar experiments with an assistant [1].

In the years 1915-1921 Born did research on a variety of topics, including chemistry (Born-Haber cycle). I don't have the time to go into the details.

Born did not participate in the development of "old quantum theory" before he came to Göttingen in 1921. Here a very short review:
In order to understand the stability of atoms and to present a theoretical description for the spectral lines emitted by hydrogen at high temperatures Niels Bohr (1885-1962) *postulated* in 1913 that the electron does *not* radiate on *stationary orbits* for which the angular momentum takes values given by *integer multiples* of $\hbar \equiv h/2\pi$. This assumption leads to discrete energy values $E_n \equiv W(n) = -E_R/n^2$ with integer n and the Rydberg energy E_R. Further postulating that in the tran-

sition of the electron from orbit with quantum number m to n a light quantum of frequency $\nu_{mn} = (E_m - E_n)/h$ is emitted, Bohr was able to explain experimental hydrogen spectra like the Balmer series [3]. Despite the "ad hoc" character of his "rules" his work attracted enormous attention.
Bohr's approach was generalized by Arnold Sommerfeld with the quantization of the classical action for multiple periodic systems in 1915 and in 1916 he presented the energy of a hydrogen-like atom with one electron and nuclear charge Ze within the framework of *relativistic* mechanics. In his formula the (small) *fine structure constant* $\alpha = e^2/\hbar c \approx 1/137$ provides a measure for the importance of relativistic effects. The status of the "Bohr-Sommerfeld theory" was presented by Sommerfeld in 1919 in the first edition of his book *Atombau und Spektrallinien* (English translation: Atomic structure and Spectral Lines)[4]. The experimental spectral lines of the helium atom could not be properly described by the Bohr-Sommerfeld theory. In fact, already the simplest molecule H_2^+ with a single electron presented a serious problem. It was treated by Wolfgang Pauli in his PhD thesis completed in July 1921 with Arnold Sommerfeld in Munich. Another student with Sommerfeld at that time was Werner Heisenberg.

3 The Institute of Theoretical Physics and the birth of Quantum Mechanics

When Woldemar Voigt was close to retirement Peter Debye (1884-1966) accepted a chair in 1914 to replace him. He had been assistant to Arnold Sommerfeld at the RWTH Aachen and had followed Sommerfeld to the LMU Munich in the year 1906.
In the years 1912/13 Debye and shortly later Born and von Kármán had independently presented publications on the specific heat of solids, a theoretical problem closely related to Planck's radiation formula.
Debye stayed in Göttingen until 1920 when he accepted an offer for a full professorship at the ETH Zurich. He already had been at the University of Zurich in the years 1911-1912.

3.1 Göttingen 1921-1924

The first on the list for the successor for Debye's chair in Göttingen was Arnold Sommerfeld. After he decided to stay in Munich the chair was offered to Max Born. In his negotiations he succeeded with his wish that another experimental chair for his friend James Franck was created. Born accepted the offer in 1920 and came to Göttingen in April 1921. The new Institute for Theoretical Physics officially started May 31, 1922. This is why today's celebration happens 101 years after the beginning of Max Born's third longer stay in Göttingen.

Figure 1: Max Born in the 1920th

Born's first two assistants in Göttingen were Wolfgang Pauli (1900-1958) and Friedrich Hund (1896-1997).
Wolfgang Pauli came to Göttingen in October 1921 and stayed until April 1922. His position was financed by Henry Goldman (Goldman and Sachs). In March 1922 Pauli published his thesis work about the H_2^+ molecule mentioned earlier.
Born and Pauli published one paper together on a general perturbation theory for atomic systems.
As Pauli did not like the life in a small provincial town like Göttingen he left for Hamburg to become assistant to Wilhelm Lenz known for the "(Laplace-Runge)-Lenz vector" which Lenz in 1924 used to describe a hydrogen atom in crossed external fields in the framework of "old quantum theory" [5].

Figure 2: Wolfgang Pauli in the 1920th

Friedrich Hund had studied mathematics, physics and geography and wanted to become a high school teacher. But he liked the scientific work and after getting his PhD in 1922 he became Born's assistant and participated with him in the attempts to generalize Hamiltonian mechanics to an "atom mechanics". He also intensely studied experimental atomic spectra and found the now famous "Hund's rules" in 1925.

Figure 3: Friedrich Hund in the 1920th

In June 1922 Niels Bohr gave the Wolfskehl lectures in Göttingen, later called the "Bohr-Festival" because it had started two weeks before the annual Göttin-

gen Händel-Opera-Festival. In seven lectures he presented the state of the art of the Bohr-Sommerfeld theory aiming at an understanding of atoms. In the later lectures he addressed in detail the construction of a theory of the periodic system of elements [6]. One should mention that Bohr emphasized "how incomplete and uncertain everything still is". Sommerfeld had come from Munich with his student Werner Heisenberg (1901-1976) and Pauli from Hamburg. Another young, mathematically very gifted, student who participated and played an important role later was Pascual Jordan (1902-1980). The Göttingen mathematicians David Hilbert, Felix Klein, Carl Runge and Richard Courant were also present. Among the many other prominent physicists from outside Göttingen were Paul Ehrenfest and Oskar Klein. After Bohr's third lecture Werner Heisenberg asked a question related to the quadratic Stark effect, to which Bohr gave an elusive answer [2]. This prompted a long discussion with him after the lecture, on a walk up to Göttingen's Hainberg. As a very young student Heisenberg had published a paper in which he tried to explain various atomic properties introducing half integer quantum numbers.
At the end of the lectures David Hilbert thanked Bohr that he had allowed insight into the holy grail of his scientific personality [7].
Later in 1922 Bohr received the Nobel Prize in Physics "for his investigations of the structure of atoms and of the radiation emanating from them" [8].

Heisenberg spent the winter semester 1922/23 in Göttingen while Sommerfeld was in the US during this time. After receiving his PhD in Munich with Sommerfeld in 1923 with studies on turbulence he came to Göttingen to help improve the understanding of the structure of atoms.

Figure 4: Werner Heisenberg in the 1920th

In 1924 Pascual Jordan finished his PhD work under Born's guidance on the quantum theory of radiation.

Figure 5: Pascual Jordan in the 1920th

One can summarize the situation by the end of 1924 by the statement that no real progress had been made.
Max Born's book "Vorlesungen über Atommechanik" [9] is a presentation of this "crisis". In the preface of the book finished in November 1924 Born explains that he wants to describe the limits of the present state of atomic theory. The book uses the Hamilton-Jacobi approach to mechanics and devotes a large part to atoms with a single valence electron. The problems of the old quantum theory are clearly presented.
In order to show the preliminary character of this work Born gives the book the subtitle "Volume 1". A later "Volume 2" should then be devoted to the "final atom mechanics". He calls this a daring attempt as rather little is known in this respect and it might take some years until Volume 2 would be written. Born acknowledges that many parts of Volume 1 were written by Friedrich Hund and only slightly revised by himself and the last chapter on the helium atom was devised by Werner Heisenberg.
Born was right with his estimate for the appearance of Volume 2 (1930), but wrong about the time of the breakthrough. The paradigm shift to quantum mechanics took place already within the next two years.

In the preface of the book Born points out the fact that for the radioactive decay only a probabilistic description is possible, a concept which should also be used for atomic transitions. This was first proposed by A. Einstein in his paper "On the quantum theory of radiation" [10], where he introduced probabilities per unit time for the transition between two atomic stationary states.

3.2 1925: Matrix Mechanics and beyond

The term *Quantenmechanik (quantum mechanics)* was first used by Max Born in the paper "Über Quantenmechanik" [11] in 1924. It was another attempt towards the "final atom mechanics", but without a breakthrough.
Heisenberg spent the winter 1924/25 in Copenhagen working with Nils Bohr and Hendrik Kramers.

Göttingen:

Heisenberg was back in Göttingen in April 1925. After a strong attack of hay fever in early June he retreated to the treeless island Helgoland in the North Sea for two weeks. He had started his attempt to formulate a completely new quantum mechanics. Instead of the hydrogen atom he addressed the anharmonic oscillator. On Helgoland he made progress with his new approach, especially concerning energy conservation. Back in Göttingen he started to write his paper "Über quantentheoretische Umdeutung kinematischer und mechanischer Beziehung (Quantum theoretical re-interpretation of kinematic and mechanical relations)"[12].
Between June 21 and July 9 Heisenberg sent four letters to Pauli which clearly show the ups and downs of his feelings about his achievements [13].
On July 9 Heisenberg sends his manuscript to Pauli asking for critical remarks. He writes:
"I am convinced about the negative critical part, but the positive one I judge as rather formal and meager, maybe more gifted people are able to make something reasonable out of it."
As Pauli apparently had no objections Heisenberg gave the paper to Born in mid July and asked him to submit it to "Zeitschrift für Physik" in case its content would make sense to him and left for Munich and a hiking tour in the Alps. A few days later Born read the paper and as he was impressed (see below) submitted it to *Zeitschrift für Physik*.

Heisenberg begins his paper by stating that he wants to lay the foundations of a quantum theoretical mechanics in which only relations between observable quantities occur. A similar statement can be found in an earlier paper by Born and Jordan on the quantum theory of aperiodic processes, received by Zeitschrift für Physik June 11, 1925 [14].

Heisenberg considers e.g. the position and the period of the electron motion as unobservable. Considering the radiation emitted by an atom he points out the importance of the associated Einstein-Bohr transition frequencies

$$\nu(n, n-\alpha) = \frac{1}{h}[W(n) - W(n-\alpha)]$$

with integer α and the $W(m)$ are the energies of Bohr's stationary orbits. Using Newton's equations of motion Heisenberg treated the one-dimensional anharmonic oscillator where the electron undergoes a periodic motion labeled by quantum number n. The classical coordinate $x(n,t)$ can then be described by a Fourier series. In order to describe the radiative transitions Heisenberg proposed to replace the Fourier coefficients by quantities $X(n, n-\alpha)$ depending on the *two* quantum numbers n and $n-\alpha$, like the transition frequencies $\nu(n, n-\alpha)$.
Focussing the description on pairs of states and their transition amplitudes had already been done in the paper by Born and Jordan [14] mentioned above. They argued that the transition probability between the different states is determined by the absolute square of the amplitudes.
The really bold step in Heisenberg's paper concerned the question: which quantum object $Y(t)$ corresponds the classical quantity $x(t)^2$? Arguing with the Ritz combination principle Heisenberg comes up with his multiplication rule for transition amplitudes

$$Y(n, n-\beta) = \sum_{\alpha} X(n, n-\alpha) X(n-\alpha, n-\beta) \ .$$

which he considers the "simplest and most natural assumption". A few steps later he points out that while classically $x(t)y(t)$ always equals $y(t)x(t)$ this is not necessarily the case in quantum theory. With the realization that quantum theory has to deal with possibly *noncommuting mathematical objects* quantum mechanics was born. But it took some time until this was formulated in the form students learn it today. No further details of Heisenbergs paper are discussed here. There is general agreement that Heisenberg's paper is notoriously difficult to read [15]. Nobel Prize winner Steven Weinberg writes in his book *Dreams of a Final Theory*:
"I have tried several times to read the paper that Heisenberg wrote on returning from Helgoland, and, although I think I understand quantum mechanics, I have never understood Heisenberg's motivation for the mathematical steps in his paper....Heisenberg's paper was pure magic".

It took Born about a week to realize that Heisenberg's multiplication rule was nothing but the multiplication rule for *matrices* he had learned as a student in Breslau. In matrix language Heisenberg had only worked with the diagonal element of the commutator of position q and momentum p. Born "easily guessed" the off-diagonal elements and was the first to obtain the basic commutation relation [15]

$$pq - qp = \frac{h}{2\pi i} \; .$$

The concept of matrices was unknown to Heisenberg when he wrote his paper. This would have been different by having a look at the book "Methoden der mathematischen Physik I" published by Richard Courant and David Hilbert in 1924, mostly written by Courant. It was different with Pascual Jordan. He had been one of Courant's assistants in the preparation of this book and was therefore familiar with matrix algebra.

As Heisenberg was not in Göttingen, Born together with Pascual Jordan took a closer look at a proper "derivation" of his guess. Stimulated by Heisenberg's paper they began to formulate *matrix mechanics* for systems of a single degree of freedom. The fact that the infinite matrices for q and p are mathematically rather subtle objects was not taken very seriously. Their paper "Zur Quantenmechanik (On Quantum Mechanics)" was received by *Zeitschrift für Physik* September 27, 1925 [16]. In this paper the commutation relation of the matrices for position and momentum appeared in print for the first time and is called "the stronger quantum condition". All further conclusions are based on it.
Heisenberg was in Copenhagen in September and therefore had been unavailable for discussions. Born had informed Heisenberg about his collaboration with Jordan. Heisenberg was excited about the achievements of his colleagues and began to work on matrix mechanics himself in Copenhagen after making himself familiar with the mathematical concept.
By the end of September Heisenberg had come up with the commutation relations for coordinates and momenta for systems with several degrees of freedom. Heisenberg raised an objection concerning the definition of the derivative of a product of several matrices with respect to one of its factors used by Born and Jordan and proposed a different definition which is closer to the usual differentiation procedure [17].
In a letter to Pauli Heisenberg pointed out that the most important thing was still missing, the solution for the hydrogen problem within matrix mechanics. It is not discussed in the paper of the three authors Born, Heisenberg and Jordan (Dreimännerarbeit) " Zur Quantenmechanik II (On Quantum Mechanics II)" which was received by Zeitschrift für Physik on November 25 [18]. It used Heisenberg's differentiation procedure and presented the state of the art of matrix mechanics. One can find e.g. a detailed discussion of the quantum mechanical properties of angular momentum.

Hamburg:

The missing solution of the hydrogen problem was found by Pauli by the end of October after learning about the progress with the formalism of matrix mechanics from Heisenberg's letters. Proudly he reported to Heisenberg about his successful calculation of the eigenvalues of the Hamiltonian matrix of the hydrogen atom. As it did not seem possible to describe angular variables as matrices Pauli wrote the classical Laplace-Runge-Lenz vector $\vec{A}$ [5] in matrix form

$$\vec{\mathbf{A}} = \frac{1}{Ze^2 m_e} \cdot \frac{1}{2} \left(\vec{\mathbf{l}} \times \vec{\mathbf{p}} - \vec{\mathbf{p}} \times \vec{\mathbf{l}} \right) + \frac{\vec{\mathbf{r}}}{r}$$

and showed that its components are constants of motion in the Coulomb potential Ze^2/r, as in the classical case. With the help of the commutation relations of the components of the vector matrix $\vec{\mathbf{A}}$ among each other and the components of the angular momentum matrix $\vec{\mathbf{l}}$, Pauli obtained Bohr's energy values $W(n)$ after an algebraic tour de force. Therefore it is usually not presented in quantum mechanics textbooks. Pauli submitted this important missing piece for the success of of matrix mechanics only much later (received by *Zeitschrift für Physik* 17 January, 1926) [19], because he wanted to include relativistic corrections. It was almost a bit too late, as discussed in the next section.

Cambridge:

The theoretical physicist who had no closer contact to the Göttingen physicists and made essential contributions to the algebraic formulation of quantum mechanics was Paul Adrien Maurice Dirac (1902-1984) in Cambridge. On July 28 Heisenberg had given a talk in Cambridge but only mentioned his new work to Ralph Fowler after the talk. Dirac received a copy of the proof-sheets of Heisenberg's paper from Ralph Fowler in early September 1925, about a month before it appeared in print. The paper made no easy reading for Dirac.

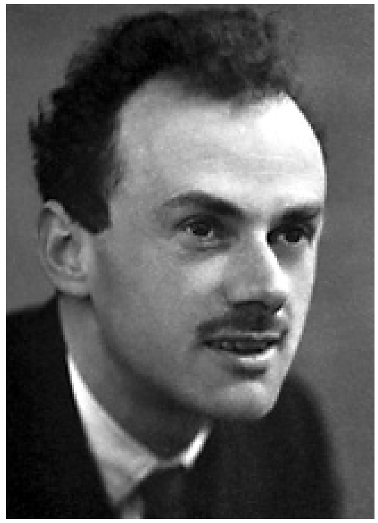


Figure 6: Paul Adrien Maurice Dirac in the 1920th

Probably by end of September he realized that most of the quantum mechanical equations can be written in a form similar to classical Hamilton mechanics using *Poisson brackets*. For differentiable functions A and B of the canonical variables they are defined as (Dirac writes [,] instead of { , })

$$\{A,B\} \equiv \sum_{i=1}^{n} \left(\frac{\partial A}{\partial q_i}\frac{\partial B}{\partial p_i} - \frac{\partial A}{\partial p_i}\frac{\partial B}{\partial q_i} \right) ,$$

which e.g. implies $\{q_i, p_j\} = \delta_{ij}$. In his contribution "The fundamental equations of quantum mechanics", received by *Proceedings of the Royal Society* on November 7 [20] Dirac writes after looking at the behaviour of Heisenberg's $Y(n, n-\beta)$ for the product of two different quantum quantities in the limit of large n and small β: "We make the fundamental assumption that *the difference between the Heisenberg products of two quantum quantities is equal* $ih/2\pi$ *times their Poisson bracket expression*."
Towards the end of the paper Dirac writes down what is usually called *Heisenberg's equation of motion*, despite the fact that it is not in Heisenberg's paper discussed above.

For the winter semester 1925/26 Born was granted leave of absence in Göttingen in order to present lectures and talks in the United States. There he received a reprint of Dirac's paper and was totally suprised to see this important contribution by a young physicist he had not heard of before.

3.3 1926: Wave Mechanics and beyond

In Zurich Erwin Schrödinger (1887-1961), born in Vienna, arrived at a new quantum theory on a completely different way. His starting point was the 1924 PhD thesis of Louis de Broglie (1892-1987) extending the wave-particle dualism of light to particles with a nonzero rest mass m. He postulated that such a particle, like an electron, also has wave character, with the wave length $\lambda = h/p$ determined by the momentum p of the particle. The issue how Schrödinger arrived at his *wave equation* will not be discussed here. In his first paper "Quantisierung als Eigenwertproblem (Quantization as an eigenvalue problem)" received by Annalen der Physik January 27, 1926 [21] he presented the (time independent) Schrödinger equation for the complex wave function ψ for a particle in an external field. This first paper in a series of papers focuses on the solution of the hydrogen problem.

Figure 7: Erwin Schrödinger in the 1920th

Early in 1926 there seemed to exist two different theoretical approaches to explain atoms: matrix mechanics and wave mechanics. But rather quickly Schrödinger (and others) showed the complete equivalence of the two approaches to (non-relativistic) quantum mechanics [22]. Schrödinger's wave mechanics quickly found more acceptance than matrix mechanics had. It was closer to the mathematics known to theoretical physicists from other fields of physics. Now external potentials different from the $1/r$ Coulomb potential could be successfully treated. This was not the case for matrix mechanics which was considered "difficult". In Munich Sommerfeld proclaimed "Wir glauben an Heisenberg, aber wir rechnen nach Schrödinger" (we believe in Heisenberg, but calculate according to Schrödinger).

The "three men" Göttingen group was not happy about this, especially as the

physical meaning of the wave function was not generally agreed on. Schrödinger had a "smearing" of the electron in mind. This question brought Max Born back into the game. As he was unable to describe the scattering of particles within matrix mechanics he successfully used the Schrödinger equation [23] for that purpose. He realized that Schrödinger's quantum mechanics can describe scattering events, but not what *definitely* happens, only how *probable* the effect is. In a footnote of his paper he presented *his* interpretation of the wave function of a particle: $|\psi(\vec{x})|^2 \Delta V$ determines the *probability* to find the particle in the (small) volume ΔV around the position $\vec{x}$.
Born reported about his switching to wave mechanics in a meeting of the Göttingen Academy of Sciences on January 14, 1927 [24]. In his introduction he states (my translation):
"While for periodic systems the wave mechanical description of the quantum laws according to *Schrödinger* provides nothing more and nothing less than the matrix representation of *Heisenberg, Jordan and myself*, it seems especially well suited for aperiodic processes. But it is *necessary* to drop completely Schrödinger's ideas which are heading towards a revival of classical continuum theory. One only has to take the formalism and give it a new physical content. One has to assume the existence of a *guiding field* which determines the probability of discrete elementary acts. As shown recently one can get the laws for the scattering of point particles (electrons, α-particles) off atoms this way".
In the last sentence Born referred to his own paper [23]. His statement nicely describes his mixed feelings about the events. Despite the fact quantum mechanics had started in Göttingen about half a year before Schrödinger, his approach was more openly accepted by the community.

By the end of 1926 Dirac and Jordan independently submitted papers on "transformation theory" which gave a general formal framework for (non-relativstic) quantum mechanics [25,26].

Another important event for physics in Göttingen should be mentioned. In Dezember 1926 James Franck shared the 1925 Nobel Prize in Physics with Gustav Hertz *... for their discovery of the laws governing the impact of an electron upon an atom.* Their scattering experiments in 1914 of electrons off mercury atoms could be interpreted as the atoms having discrete energy states. This fitted well to Bohr's ideas.
In their common Goettingen years Max Born had regular discussions with James Franck about his experimental activities [1].

3.4 Interpretation, Applications and Nobel Prizes

While the formalism of non-relativistic quantum mechanics was ready by the end of 1926 its interpretation was still in its infancy. In March 1927 Heisenberg submitted a paper with his famous uncertainty relation [27]. First leaving aside the experimental implications it is a mathematical result for the square root of the variances $(\Delta x)_{|\psi\rangle}$ and $(\Delta p)_{|\psi\rangle}$ of the probability densities for the position and momentum of a particle in the state $|\psi\rangle$

$$(\Delta x)_{|\psi\rangle}(\Delta p)_{|\psi\rangle} \geq \hbar/2$$

which can be derived using the commutation relation for position and momentum. Discussions of Heisenberg with Bohr, Pauli and others about this inequality, Born's probability interpretation of the state and the proposal of the "collapse of the wave function" by a measurement with a macroscopic experimental device [28] led to what is usually called the *Copenhagen interpretation* of quantum mechanics, despite the fact that there is no general agreement about its concise meaning.
Einstein and Schrödinger were the most famous opponents of the Copenhagen interpretation.
Let me mention a statement by John Bell who also saw the Copenhagen interpretation rather critical. In his article "Against 'measurement' " he nevertheless writes:
"ORDINARY QUANTUM MECHANICS (as far as I know) IS JUST FINE FOR ALL PRACTICAL PURPOSES (FAPP)" [29].
The discussion of the many alternative interpretations continue till today and would be a topic for a separate talk.

Soon after its formulation, quantum mechanics was successfully applied to various problems in atomic-, molecular-, nuclear- and solid state physics, which can only be touched upon here. Again the focus is on the Göttingen players. Friedrich Hund discovered the phenomenon of "tunneling" in quantum mechanics and before Robert Mullikan showed the importance of what was later called "molecular orbitals" [30]. Also Born together with Robert J. Oppenheimer addressed problems in molecular physics. The much larger nuclear mass compared to electron mass led to the formulation of the "Born-Oppenheimer approximation" which lies at the heart of chemistry [31]. Dirac spent the first half of 1927 in Göttingen and lived in the same house "Am Geismartor 4" as Oppenheimer. Other PhD students or visitors with Born were e.g. Lothar Nordheim, Max Delbrück, Victor Weisskopf, Maria Göppert, Walter Heitler, Vladimir Fock and Edward Teller. One should also mention John von Neumann and Eugene Wigner, who were closer to David Hilbert. It was nice but also strenous for Born to follow the activities of all these gifted young scientists. In 1928/29 Born was close to a nervous breakdown and spent some time at a sanitorium at Lake Constance (Bodensee).

A very large number of Nobel Prizes was awarded for work related to quantum mechanics, especially for various applications. Three of the five authors who formulated quantum mechanics in 1925/26 were awarded the Nobel Prize in Physics in 1933. Werner Heisenberg received the 1932 prize for “for the creation of quantum mechanics, the application of which has, inter alia, led to the discovery of the allotropic forms of hydrogen”[7]. Erwin Schrödinger and Paul Dirac shared the 1933 prize “for the discovery of new productive forms of atomic theory”[7]. Here I discussed Dirac’s contribution to the formulation of nonrelatistic quantum mechanics. He had made an even more important contribution with his relativistic *“Dirac equation”* for spin 1/2 particles formulated in 1928 [32]. The other two of the five “fathers” of quantum mechanics, especially Max Born, were unhappy not to receive the prize. Heisenberg wrote a letter to Born in November 1933 that he is depressed to receive the Nobel Price alone [2]. Born had to wait more than twenty years and received the 1954 Nobel Prize in Physics, “for his fundamental research in quantum mechanics, especially for his statistical interpretation of the wave function”[7]. Pascual Jordan’s contribution to the formulation of quantum mechanics was not honored by a Nobel Prize.

3.5 Early Textbooks on Quantum Mechanics

The first well accepted textbook was Dirac’s “The Principles of Quantum Mechanics”[33]. Its first edition was published in 1930 and it treats the “Schrödinger representation” as well as the “Heisenberg representation”. John von Neumann two years later in the preface of his own book complained that Dirac’s book lacks mathematical rigor [34].

Dirac’s book is in sharp contrast to Born’s “Volume 2” promised in 1924 to appear probably several years later. The title of the book also published in 1930 with P. Jordan as co-author is “Elementare Quantenmechanik” [35]. The book promises the treatment of wave mechanics in “Volume 3” which was never written. Therefore “Volume 2” only treats the algebraic methods of matrix mechanics. Differential equations are avoided as consistently as possible. In a review of the book W. Pauli pointed out very clearly that he considered this restriction a bad idea [36]. Because of this one-sided approach the book could be of any use only for a very restricted readership. The only positive sentence in his review is the last one. In typical sarcastic Pauli style:

“The making of the book with respect to print and paper is excellent”.

In hindsight one has to admit that this book definitely did not help to popularize the important first steps towards nonrelativistic quantum mechanics made in Göttingen. Much later Born admitted that Pauli was completely right [2].

4 The later years of Max Born

When national socialism came to power the role of Göttingen as one of the centers for quantum physics ended abruptly.
James Franck openly protested against the suspension of non-Aryan civil servants in April 1933. He emigrated with his family to Baltimore where he was offered a professorship at Johns Hopkins University. Like Born he was of Jewish descent. Franck would have been exempt from the suspension as he had fought for Germany in World War I (*Frontkämpfer*).

In July 1933 Born applied for a three year leave of absence to accept a lectureship at the University of Cambridge. It was granted in October 1933 and his suspension was canceled. With his wife Hedi and his children Irene, Margarete and Gustav he moved to Great Britain.

In the winter 1935/36 Born spent six months in Bangalore (India) working with C. V. Raman.

End of 1935 Born was informed about the early retirement on his Göttingen professorship. In 1936 the Ministry in Berlin ordered Richard Becker to follow Born from Berlin (Technische Hochschule, TU since 1946) on his Göttingen chair. As Becker had had friendly exchanges with Born about various topics (e.g. the cohesion of single crystals), he was not happy to be forced to take Born's chair in Göttingen.

In the same year Born accepted the Tait-Professorship for Natural Philosophy in Edinburgh.

In November 1938 the Born family was deprived of their German citizenship. The Borns received the certificate of naturalisation as British subjects the day before the outbreak of World War II .

In 1953 Max Born, James Franck and Richard Courant were named honorary citizens of Göttingen. As a professor emeritus Born returned to Germany and lived in Bad Pyrmont about fifty miles northwest of Göttingen from 1954 up to his death in 1970. There in early November 1954 he received the message, that he would share the 1954 Nobel Prize in Physics with Walter Bothe, the latter for coincidence methods in nuclear physics.

After Richard Becker's death in 1955 Friedrich Hund accepted his chair in the Institute of Theoretical Physics and came back to Göttingen in 1957 from Frankfurt (Main).
In the same year Born signed the "Göttinger Manifest" against arming the Ger-

man Bundeswehr with nuclear weapons.

From Bad Pyrmont Born visited Göttingen often. Let me mention a few occasions:

At the opening ceremony of the building enlarging the Institute of Theoretical Physics at the Bunsensstrasse in 1957 Born was present and gave a lively report about his three periods in Göttingen.

In 1959 Born gave a copy of of his new book with co-author Emil Wolf "Principles of Optics" to the Institute with a handwritten note:
Dem Institut für Theoretische Physik der Universität Göttingen von dem einstigen Leiter Max Born
This is a most influential book, a classic science book of the twentieth century.

At the occasion of Born's 80-th birthday a special Colloquium was held at the big lecture Hall at the Bunsenstrasse, with Werner Heisenberg presenting the talk "Quantenmechanik und die Theorie der Elementarteilchen".
Born was also present when Friedrich Hund celebrated his seventieth birthday in 1967. Heisenberg had come from Munich.

Figure 8: Friedrich Hund's 70-th birthday

Max Born died on January 5, 1970 in Göttingen. On his gravestone at the Göttingen Cemetery (Stadtfriedhof) "his" commutation relation can be read.

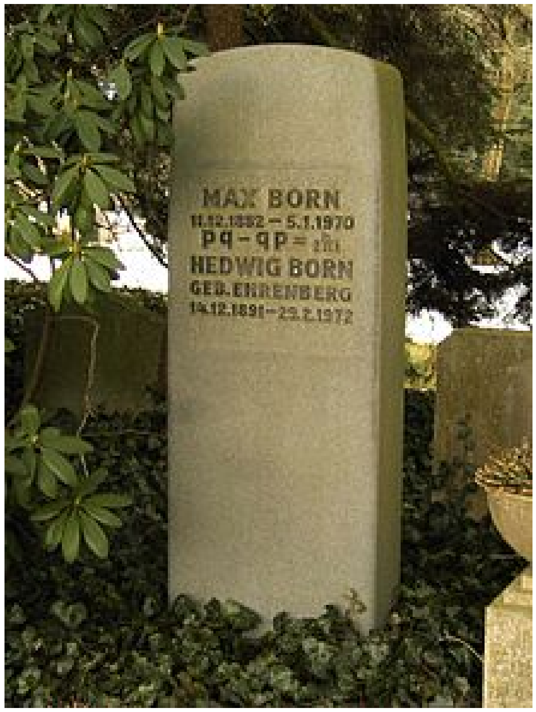
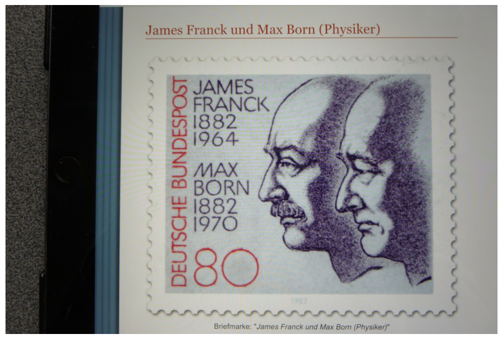


Figure 9: Born's gravestone

A short side remark: Erwin Schrödinger died nine years before Max Born. In the upper part of his grave cross in Alpbach in the Austrian alps one can read the time dependent Schrödinger equation: $i\hbar\dot{\psi} = H\psi$.

Born is also present in "his" Institut für Theoretische Physik with a bust created by Margarete Autschbach in the nineteen sixties.

We are here because of the hundredth birthday of the ITP. The hundredth birthdays of the two physicists central for its establishment, Max Born and James Franck were honoured in 1982 by a stamp of the *Deutsche Bundespost*.



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