UNIVERSITÄT DES SAARLANDES

Strongly disordered quantum magnets

- Ising chain with random bonds and transverse fields
- Dasgupta-Ma renormalization group (SDRG)
- Infinite randomness fixed point (IRFP)
- Griffiths-McCoy singularities
- Random transverse Ising systems (d>1, FM and SG)
- Random Heisenberg quantum magnets (d>1, AFM & SG)
- Dynamics, Quantum FDT and its violation

Göttingen, 4.11.2003

Collaborators:

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N. Kawashima (MU Tokio)

Random transverse Ising chain

$$H = -\sum_i J_i \sigma_i^z \sigma_{i+1}^z + \sum_i h_i \sigma_i^x$$

$J_i \in [0,1]$ Random bonds

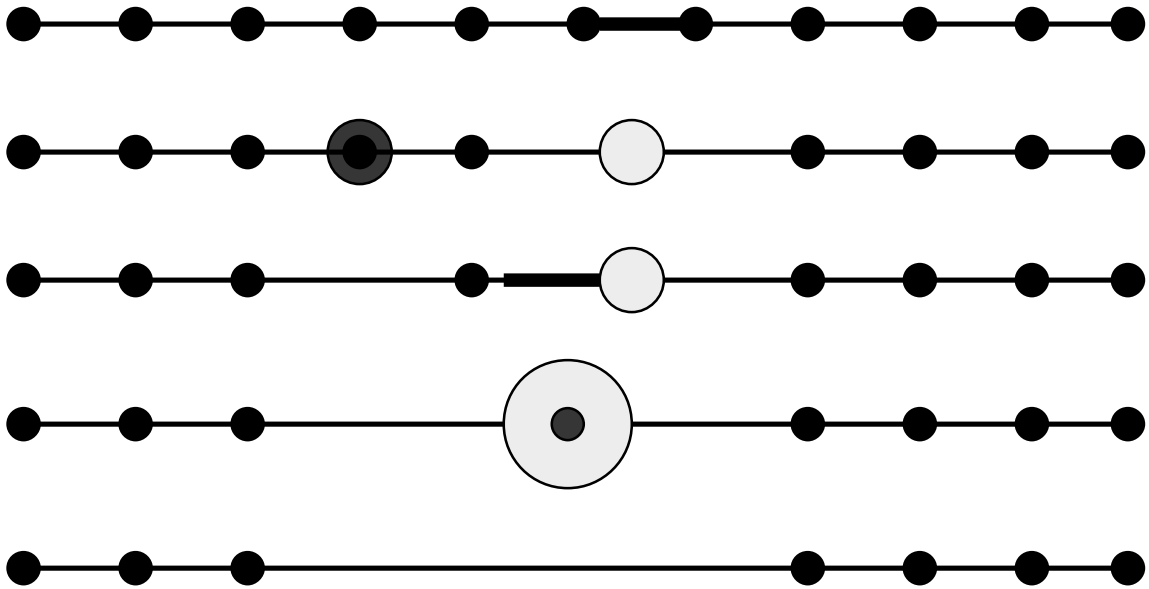
$h_i \in [0,1]$ Random transverse fields

Paramagnetic at all temperature T>0

Quantum phase transition (QPT) at T=0 when $[\ln h]_{av} = [\ln J]_{av}$

Distance from the QCP: $\delta = [\ln h]_{av} - [\ln J]_{av}$

RG procedure – sketch:

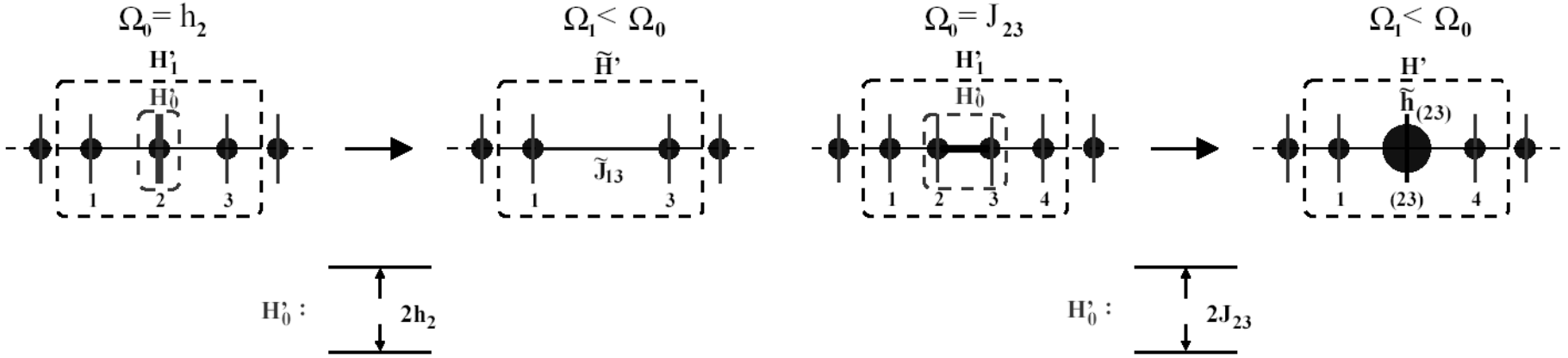


Etc.
To be continued until all bonds / fields are decimated

Ma-Dasgupta-Hu (MDH) Renormalization Group

Strategy: Find strongest coupling Ω_0 (J_{ij} or h_i) and minimize this term in the Hamiltonian.

[Ma, Dasgupta, Hu '79; Dasgupta, Hu '80; D. Fisher '92]



$$H_0' = -h_2\sigma_2^x$$

$$H_1' = -J_{12}\sigma_1^z\sigma_2^z - J_{23}\sigma_2^z\sigma_3^z$$

$$\Rightarrow \quad \boxed{\tilde{H}' = -\tilde{J}_{13}\sigma_1^z\sigma_3^z}$$

with: $\boxed{\tilde{J}_{13} \approx \frac{J_{12}J_{23}}{h_2}}$

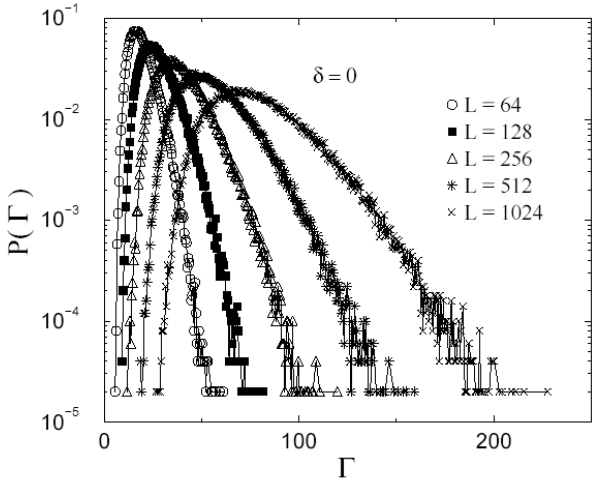
$$H_0' = -J_{23}\sigma_2^z\sigma_3^z$$

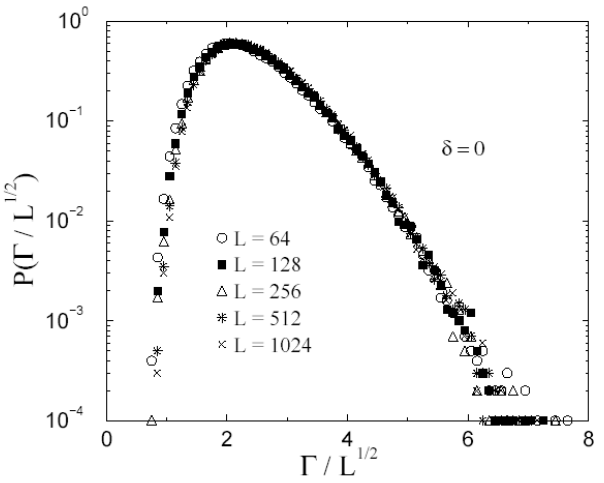
$$H_1' = -h_2\sigma_2^x - h_3\sigma_3^x$$

$$\Rightarrow \quad \boxed{\tilde{H}' = -\tilde{h}_{(23)}\sigma_{(23)}^x}$$

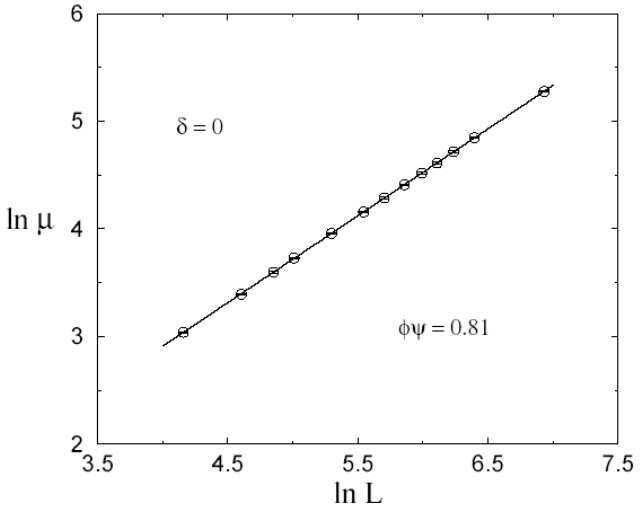
with: $\boxed{\tilde{h}_{(23)} \approx \frac{h_2h_3}{J_{23}} \; ; \quad \tilde{\mu}_{(23)} = \mu_2 + \mu_3}$

Numerics: Results of the RG (1d)

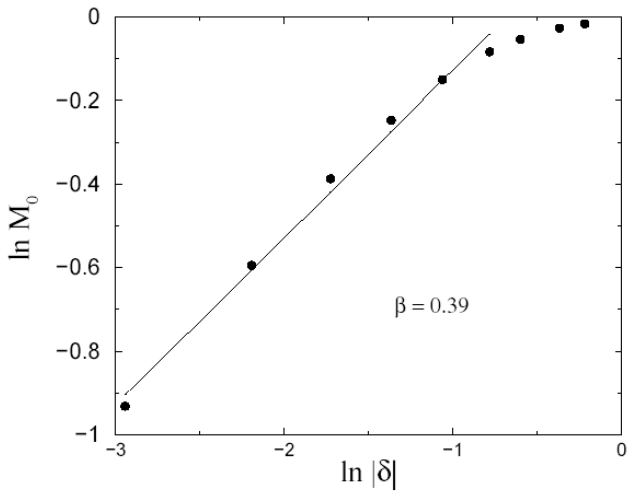




Probability distribution of the log of the last field to be decimated



FSS of the # of spins in the last cluster



Mag. moment in in the FM phase

Analytical , random walk picture

During the RG process:

$$J' = \frac{J_1 J_2}{h_1}, \quad h'' = \frac{h'_1 h'_2}{J'_1}, \quad h''' = \frac{h''_1 h''_2}{J''_1}, \quad etc.$$

$$x \sim \prod_i y_i \Rightarrow \ln x \sim \sum_i \ln y_i \qquad y_i = (h_{j(i)} / J_{k(i)})^{\pm 1}$$

c.f. exact result for surface mag.:

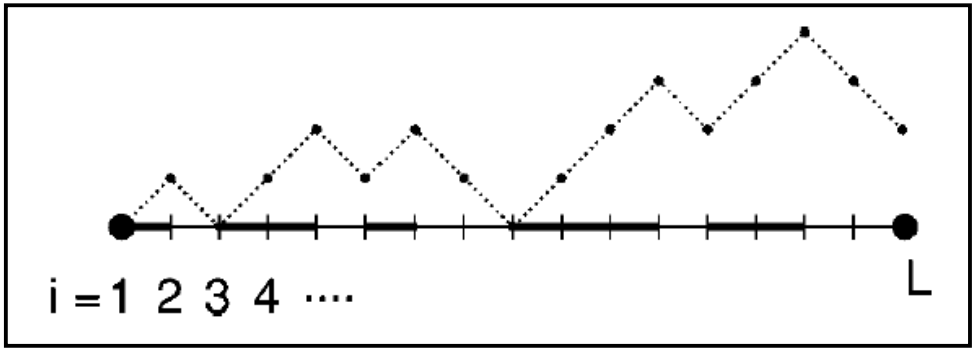
$$m_s = \left(1 + \sum_{i=1}^{L-1} \prod_{j=1}^{i} \frac{h_j}{J_j} \right)^{-1/2}$$

$m_{surface}(L) = P_{survival}(L)$ $\Leftarrow$
and much more ...

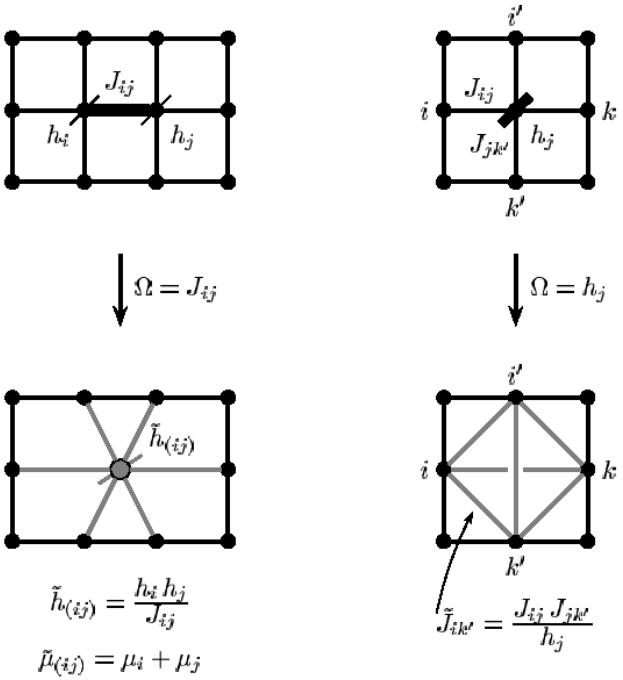
[Iglói, Rieger ´98]

Random walk picture:

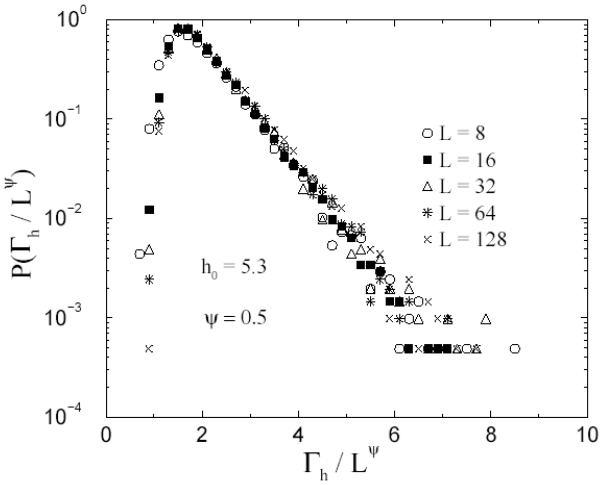
$$h_i = const.$$
$$J_i \in \{\lambda^{-1}, \lambda\}, \quad \lambda \to \infty$$



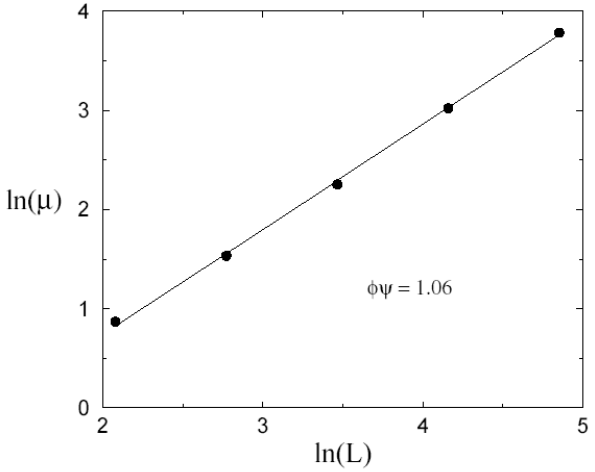
Numerics: Results of the RG (2d)



RG scheme in 2d
(n.b.: lattice structure NOT preserved)



last field distr.



Mag. moment

Lin, Kawashima, Iglói, Rieger `00

Infinite Randomness Fixed Point

Fixed point of the MDH–RG:

For random transverse Ising systems (i.e. randomly interacting TLSs):

Fixed point of the MDH–RG is completely dominated by the disorder (⇔ percolation):

Unconventional scaling with three exponents ϕ, ψ, ν

[D. Fisher '92;
Mau, Motrunich, Fisher, Huse '01]

At the critical point ($\delta = 0$):

lowest energy scale	$-\ln \Delta$	$\sim$	L^{ψ}
magnetic moment	μ	$\sim$	$(-\ln \Delta)^{\phi}$
average correlations:	$[C_{ij}]_{\mathrm{av}}$	$\sim$	$\lvert\mathbf{r}_i - \mathbf{r}_j\rvert^{-2(d-\phi\psi)}$
typical correlations:	$-[\ln C_{ij}]_{\mathrm{av}}$	$\sim$	$\kappa_{ij}\lvert\mathbf{r}_i - \mathbf{r}_j\rvert^{\psi}$
finite T-susceptibility	χ	$\sim$	$T^{-1}(-\ln T)^{2\phi-d/\psi}$
finite H-magnetization	M	$\sim$	$(-\ln H)^{-\phi+d/\psi}$

Away from the critical point ($\delta \neq 0$):

average correlation length	ξ_{av}	$\sim$	$\delta^{-\nu}$
typical correlation length	ξ_{typ}	$\sim$	$\xi_{\mathrm{av}}^{1-\psi}$
lowest energy scale	Δ	$\sim$	$L^{-z(\delta)}$
dynamical exponent	$z(\delta)$	$\propto$	$\delta^{-\psi\nu}$
finite H-magnetization	M	$\sim$	$H^{1/z(\delta)}$
spontaneous magnetization	M_0	$\sim$	$(-\delta)^{\nu(d-\phi\psi)}$

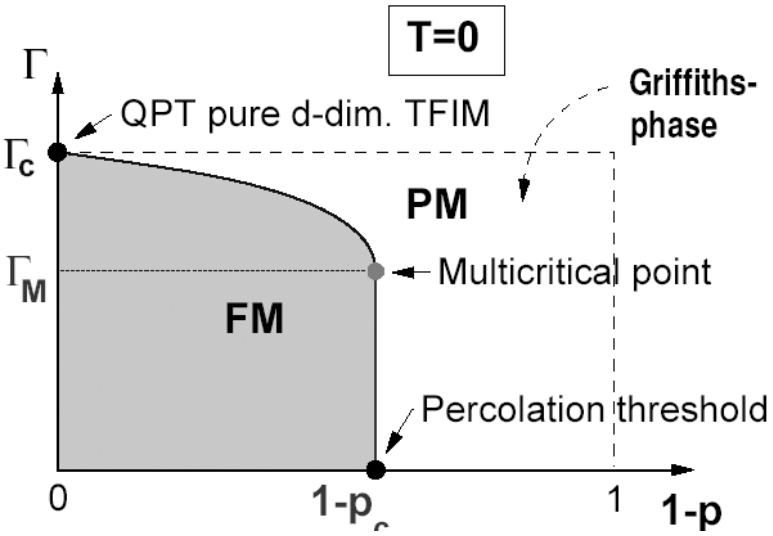
Diluted FM in d dimensions

Site–dilution: $H = -J \sum_{(ij)} \varepsilon_i \varepsilon_j \sigma_i^z \sigma_j^z - \Gamma \sum_i \varepsilon_i \sigma_i^x$

Bond–dilution: $H = -J \sum_{(ij)} \varepsilon_{ij} \sigma_i^z \sigma_j^z - \Gamma \sum_i \sigma_i^x$

$$\varepsilon_i, \varepsilon_{ij} = \begin{cases} 1 & \text{w. prob. } p \\ 0 & \text{w. prob. } 1-p \end{cases}$$

IRFP $\Leftrightarrow$ classical percolation transition: [Senthil, Sachdev '96; Ikegami, Miyashita, Rieger '98]



Classical percolation:

distance from the critical point	$\delta = p_c - p$
connectivity correlation length	$\xi \sim \lvert\delta\rvert^{-\nu_{\text{perc}}}$
fractal dimension of perc. cluster	$M \sim L^{D_{\text{perc}}}$
probability for $\sigma_i \in$ perc. cluster	$P \sim (p - p_c)^{\beta_{\text{perc}}}$

$$\Rightarrow \quad \begin{aligned} \nu &= \nu_{\text{perc}} \\ \psi &= D_{\text{perc}} \\ \phi &= (d - \tfrac{\beta_{\text{perc}}}{\nu_{\text{perc}}})/D_{\text{perc}} \end{aligned}$$

Numerics for Random Ferromagnet

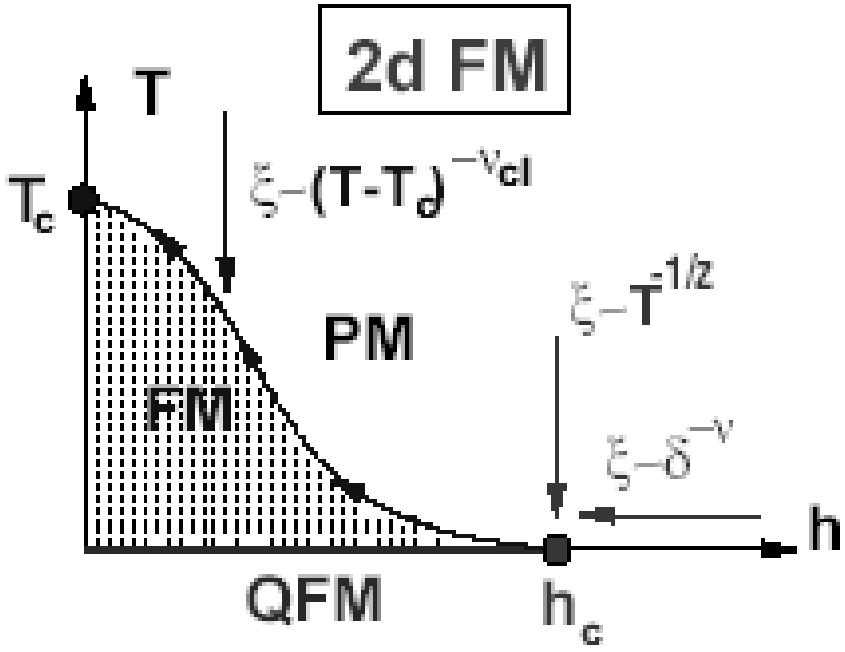
$$H = -\sum_{(i,j)\ \mathrm{n.n.}} J_{ij}\sigma_i^z\sigma_j^z - \sum_i h_i\sigma_i^z$$

$J_{ij} \geq 0$ ferromagnetic;

$$P(J_{ij}) = \begin{cases} 1, & \text{for } 0 < J < 1, \\ 0, & \text{otherwise,} \end{cases}$$

$$P(h_i) = \begin{cases} h_0^{-1}, & \text{for } 0 < h < h_0, \\ 0, & \text{otherwise.} \end{cases}$$

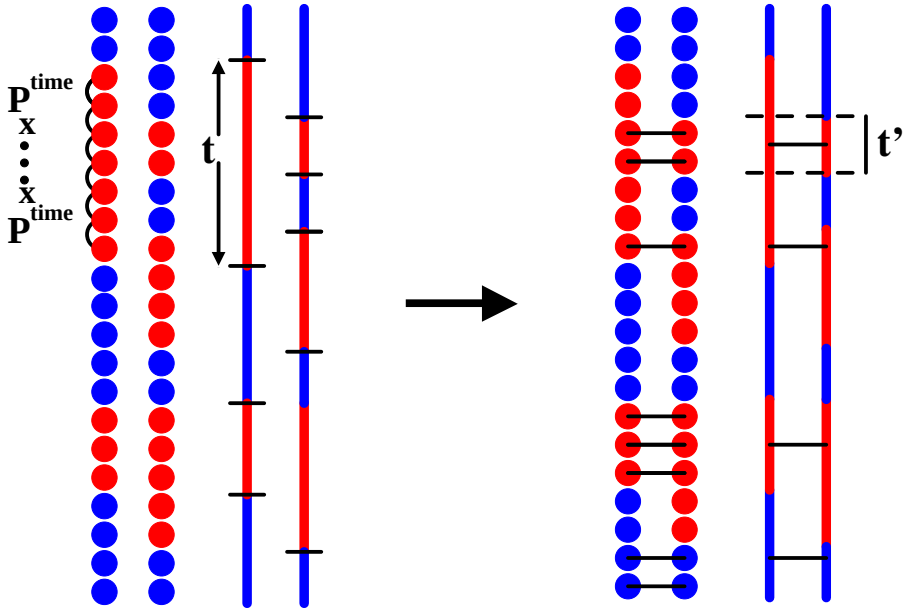
, [Pich, Young, Kawashima, Rieger '98]



QMC of d-dimensional TIM

$$H_{classical} = -\sum_{\tau=1}^{L_\tau} \left\{ \sum_{ij} K_{ij} S_i(\tau) S_j(\tau) + \sum_i K_i S_i(\tau) S_i(\tau+1) \right\}$$

$$K_{ij} = J_{ij}\,\Delta\tau, \quad \exp(-K_i) = \tanh(\Gamma_i\,\Delta\tau), \quad \Delta\tau = \beta / L_\tau$$



Anisotropic Finite-Size-Scaling

For random quantum systems:

$$\xi \sim |\delta|^{-\upsilon} \quad \text{(space)}$$
$$\xi_\tau \sim |\delta|^{-\upsilon_\tau} \quad \text{(time)}$$

$z = \nu_\tau / \nu > 1$

FSS form of a thermodynamic quantity O:

$$[O]_{av} = L^{-x_O} g_O\left(\frac{L}{\xi}, \frac{\beta}{\xi_\tau}\right)$$

Conventional dynamical scaling

$$\xi_\tau \sim \xi^z$$

$$[O]_{av} = L^{-x_O} g'_O\left(\delta L^{1/\nu}, \beta L^{-z}\right)$$

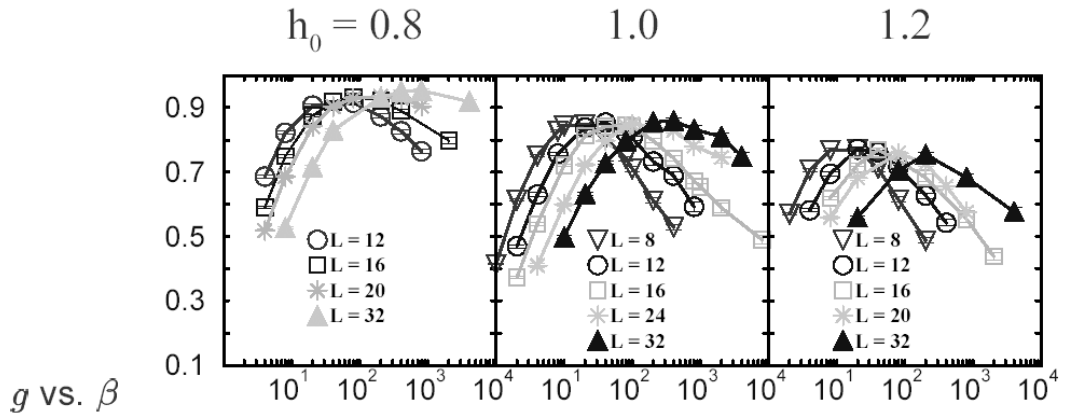
Activated dynamical scaling

$$\ln \xi_\tau \sim \xi^\psi$$

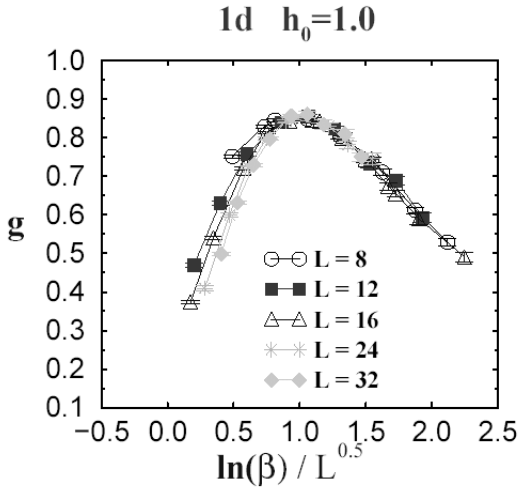
$$[O]_{av} = L^{-x_O} g'_O\left(\delta L^{1/\nu}, \ln \beta / L^\psi\right)$$

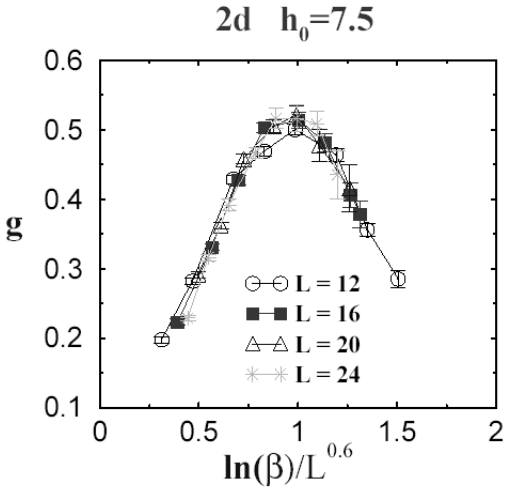
Localization of the QCP in a random FM

Example: $1d$ with uniform distributions: $J_{ij} \in [0, 1]$ $\quad h_i \in [0, h_0]$



(with continuous imaginary time QMC)





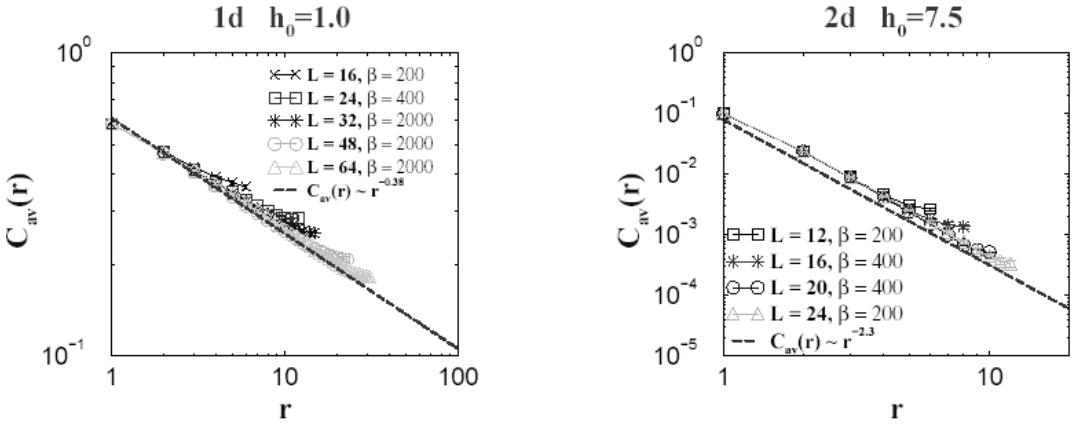
[Lin & Rieger `01]

$\Rightarrow$ Activated dynamical scaling in $1d$ and $2d$: $\ln \xi_\tau \sim \xi^\psi$

$\psi = 0.5$ for $1d$ $\quad$ & $\quad \psi \approx 0.6$ for $2d$

Average correlation function: FM (1d & 2d)

Scaling of the **average** correlation function C_{av} at the critical point

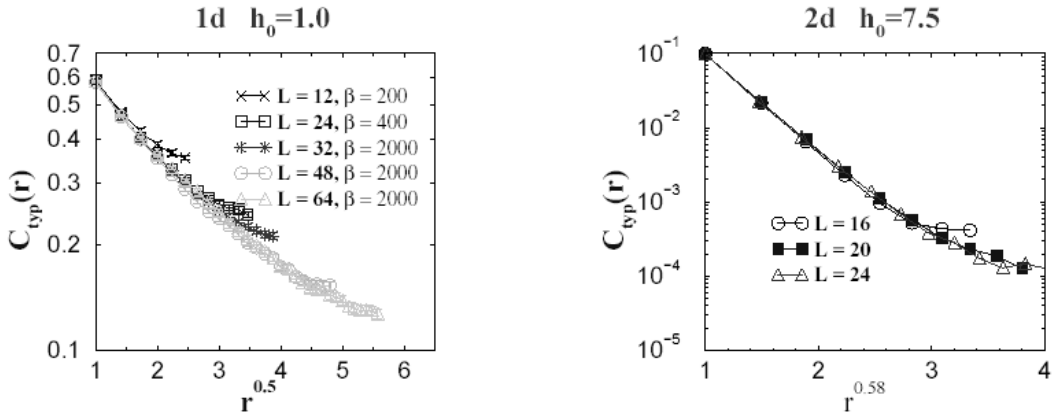


power in r: $C_{\text{av}}(r) \sim r^{-\eta}$

$1d:\ \eta \approx 0.38$ $2d:\ \eta \approx 2.3$

Typical correlation function: FM (1d & 2d)

Scaling of the **typical** correlation function C_{typ} at the critical point



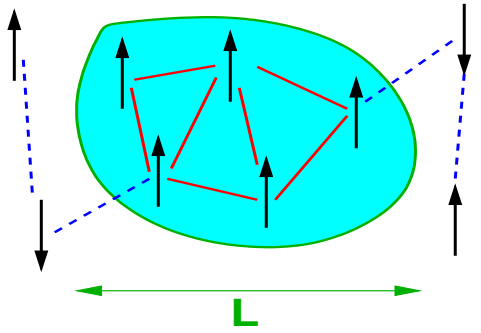
stretched exponential decay in r: $C_{\text{typ}}(r) \sim \exp(-\text{const.} r^{\psi_c})$

$1d:\ \psi_c = 0.5$ $2d:\ \psi_c \approx 0.58$

[Lin, Rieger '01]

Griffiths Mc-Coy Singularities

Phenomenological:
L: size of a region of strongly coupled spins
$P(L) \sim p^{L^d} = \exp(-\lambda L^d)$, $(p<1,\ \lambda=|\ln p|>0)$.
Tunneling gap: $\Delta_0=\exp(-sL^d)$ $(s>0)$.

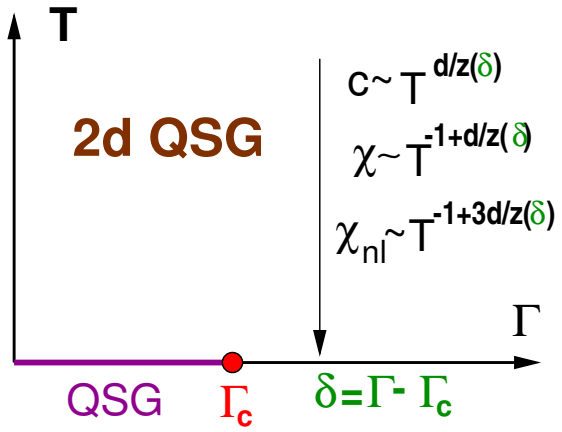


$$\Rightarrow C(\tau) = \left[\left\langle \sigma_i(\tau)\sigma_i(0)\right\rangle\right] \sim \tau^{-\lambda/s} = \tau^{-d/z(\delta)}$$

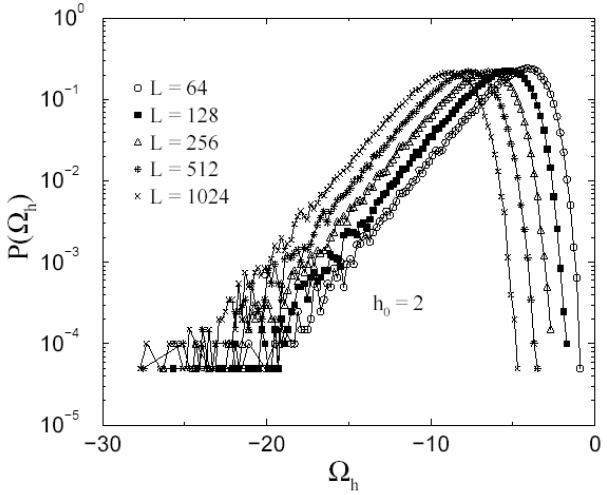
$z(\delta)=s/\lambda d$ dynamical exponent in the Griffiths phase
$\delta=(\Gamma-\Gamma_c)$ distance from the critical point

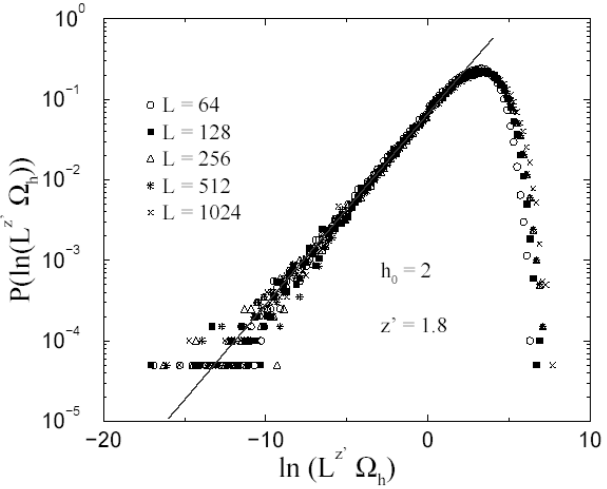
$$\Rightarrow \chi(\omega=0) = \int_0^{\beta} d\tau\, C(\tau) \sim T^{-1+d/z(\delta)}$$

In random transverse ising systems: $\lim_{\delta\to 0} z(\delta) = \infty$
Infinite Randomness Fixed Point (IRFP)

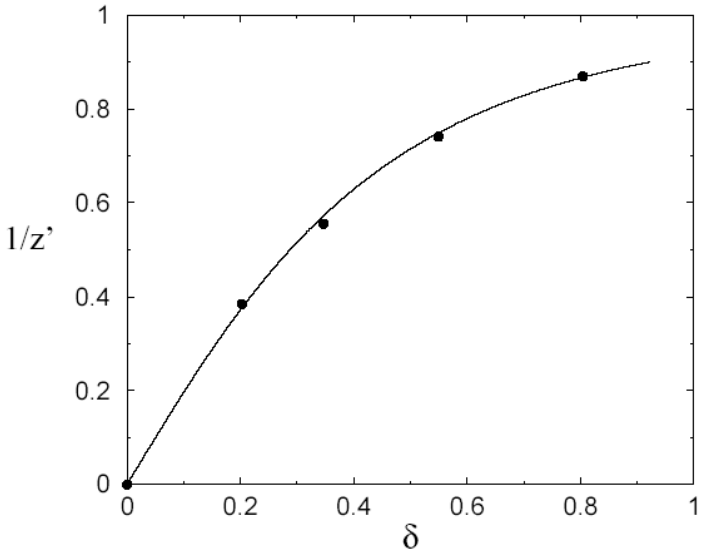


Griffiths-McCoy singularities: 1d FM



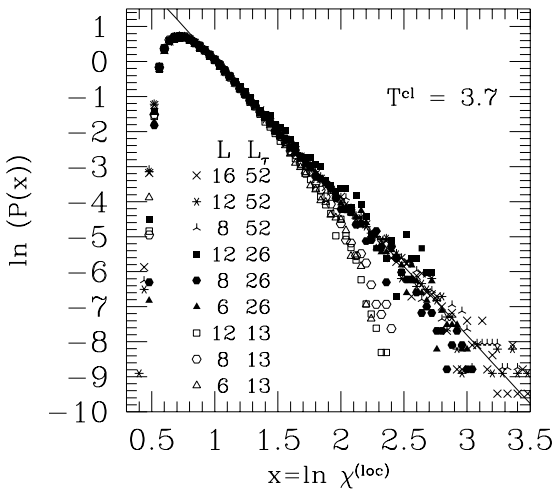


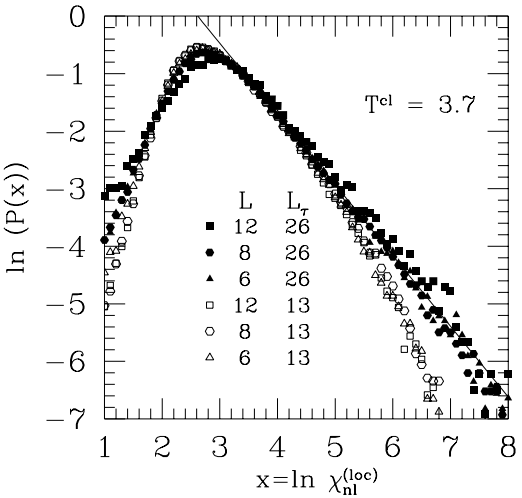
Distribution of the last log-field
Away from the critical point ($\delta>0$)



Inverse dynamical exponent z‘
as function of the distance
from the critical point
$1/z' \sim 2\delta$ for $\delta \to 0$

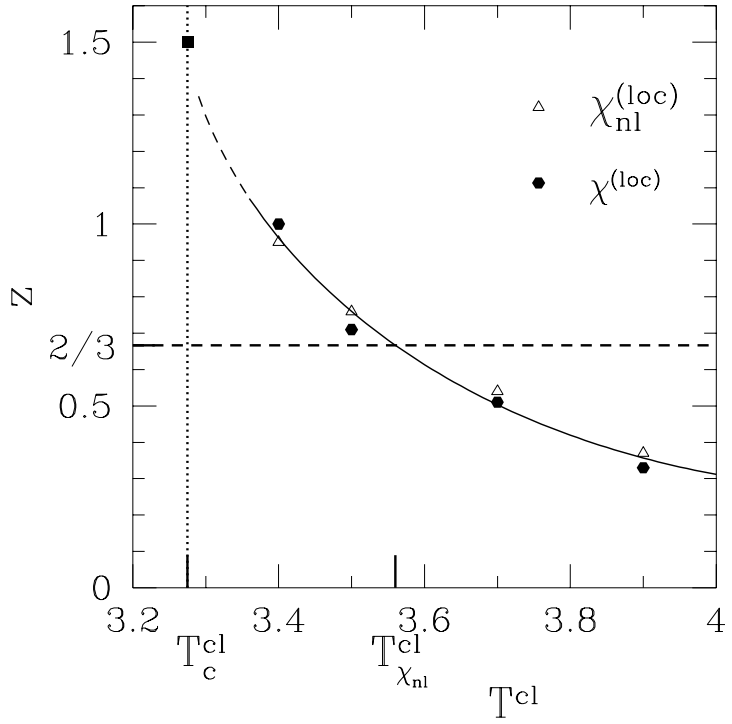
Griffiths-McCoy singularities: QSG





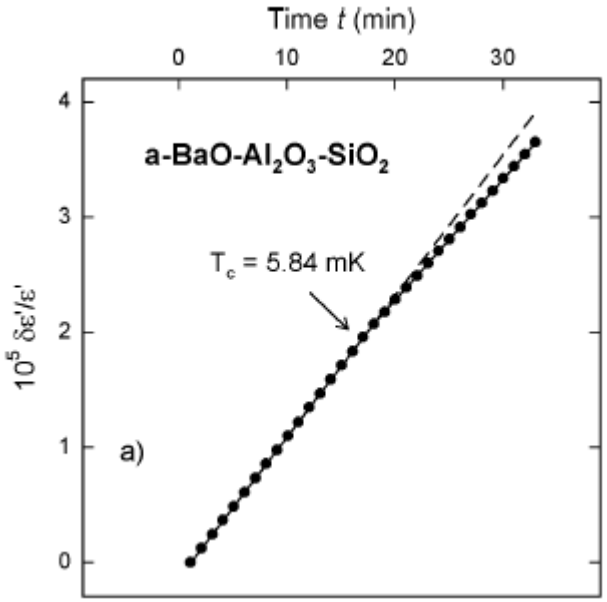
$$H = -\sum_{<ij>} J_{ij}\sigma_i^z\sigma_j^z - h\sum_i \sigma_i^x$$

J_{ij} Gaussian

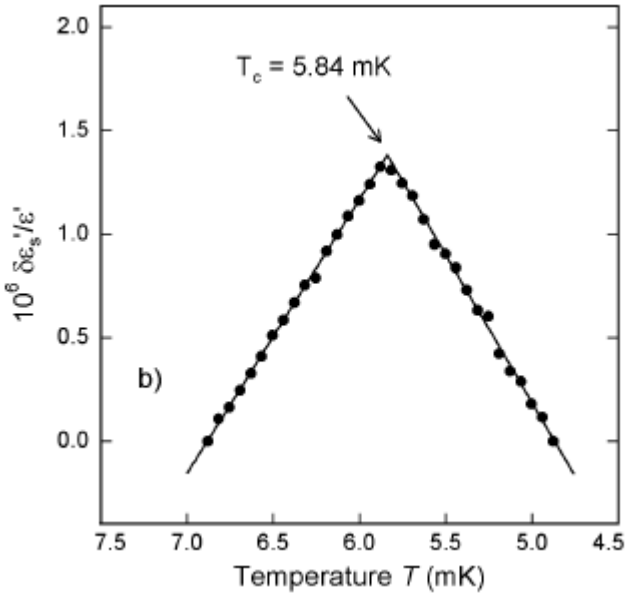


Low-T phase transition in glasses?

Dielectric constant measurement



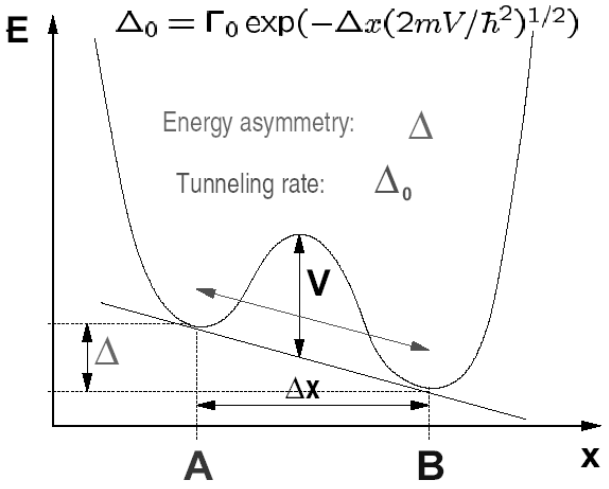
Susbstraction of the linear part



[Strehlow, Enss, Hunklinger ,97]

Low-T physics of structural glasses: Two-level systems:

Double well potential:



$\approx$ two states A and B
A$\leftrightarrow S^z = +1/2$
B$\leftrightarrow S^z = -1/2$

$\Rightarrow$ Spin-$\frac{1}{2}$ system

$$\boxed{H = -\Delta\hat{\sigma}^z - \Delta_0\hat{\sigma}^x}$$

Random energy asymmetry $P(\Delta) = const$;
random tunneling rate $P(\Delta) \propto 1/\Delta_0$;
.

$\Rightarrow$ $\boxed{C \propto T; \kappa \propto T^2}$
etc.

[Phillips '72; Anderson, Halperin, Varma '72]

Dipolar interactions due to elastic strain: $J_{ij} \sim 1/|r_i - r_j|^3$

$$\boxed{H = -\sum_{(ij)} J_{ij}\sigma_i^z\sigma_i^z - \sum_i \Delta_i\sigma_i^z - \sum_i \Delta_{0i}\sigma_i^x}$$

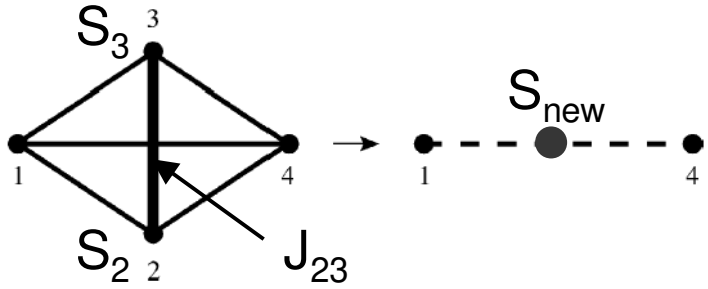
Heisenberg QSG

$$H = \sum_{(ij)} J_{ij} \vec{\sigma}_i \cdot \vec{\sigma}_j$$

J_{ij} Gaussian, zero mean, variance J

Dasgupta-Ma RG:

$$J_{23} >> |J_{12}|, |J_{13}|, |J_{24}|, |J_{34}|$$



Form a new spin S_{new} from S_2 and S_3:
$S_{new}=| S_2 \pm S_3 |$, + for F bond, - for AF bond.

For AF: $S_2 = S_3$ - singlet formation -> decimation ($S_{new}=0$),

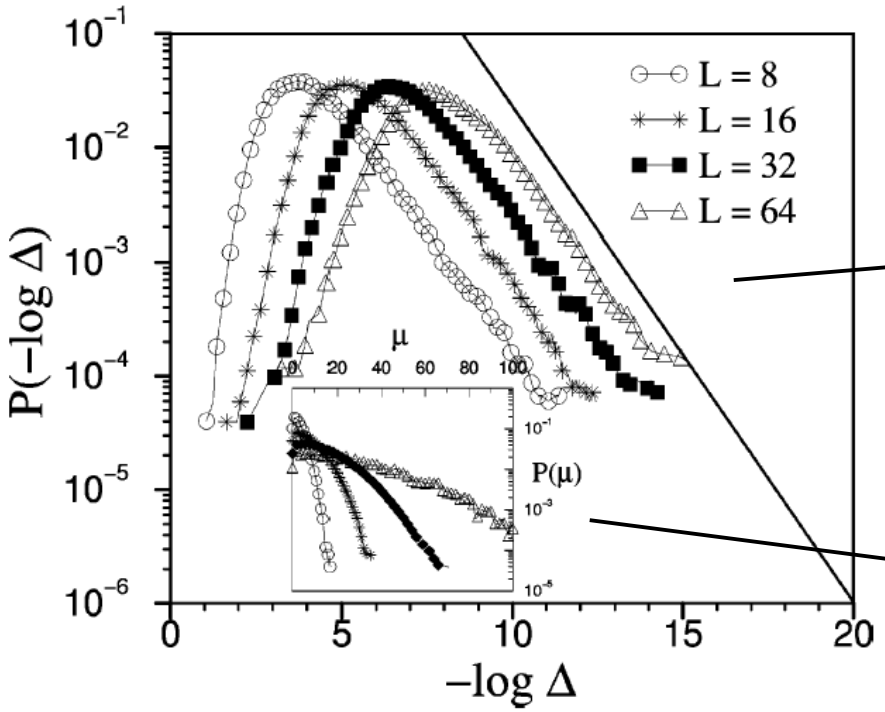
$$J_{14}^{new} = \lambda (J_{12} - J_{13})(J_{34} - J_{24}) / J_{23}$$

Otherwise:

$$J_{1,new} = c_{23} J_{12} + c_{32} J_{13} \quad and \quad J_{new,4} = c_{23} J_{24} + c_{32} J_{34}$$

$$c_{ij} = [S_{new}(S_{new}+1) + S_2(S_2+1) - S_3(S_3+1)]$$

Heisenberg QSG - 2d:



[Lin,Melin,Igloi,Rieger ,03]

Gap distribution:

$$P_L(\Delta) = L^z\, p(L^z\Delta) \sim L^{z(1+\omega)}\Delta^{\omega}$$

ω=0, z≈2

(Magnetic) moment distribution:

$$P_L(\mu), \quad \overline{\mu} \sim L^{\zeta}$$

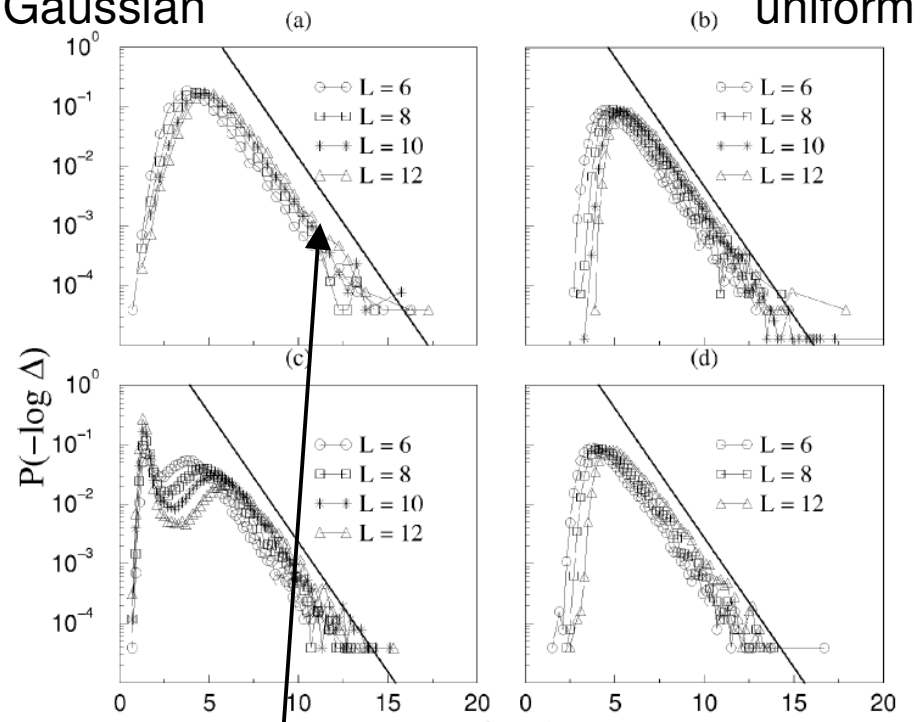
$$\overline{\mu} \sim 0.41 \cdot L \quad \Rightarrow \quad \zeta=1/2$$

If ζ>0: Large moment formation in the GS:

$$\chi(T) \sim T^{-1}; \quad c(T) \sim T^{2\zeta(1+\omega)} \left|\ln T\right|; \quad m(h) \sim h^{\frac{\zeta(1+\omega)}{1+\zeta(1+\omega)}}$$

Heisenberg QSG – 3d:

Gaussian



uniform

asymmetric uniform

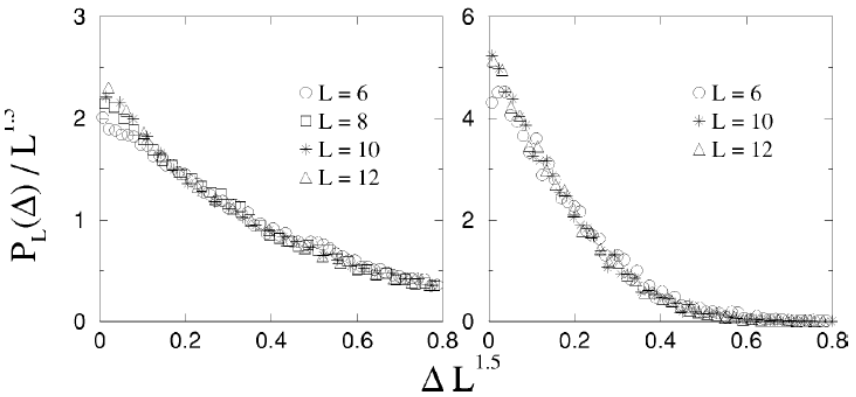
S=1 Gaussian

[Lin,Melin,Igloi,Rieger ,03]

Straight line has slope –1:

$\Rightarrow \omega=0$

Gaussian (a) uniform (b)



$$P_L(\Delta) \sim L^{1.5}\, p(L^{1.5}\Delta)$$

$\Rightarrow z = 1.5$ Note: z<d!

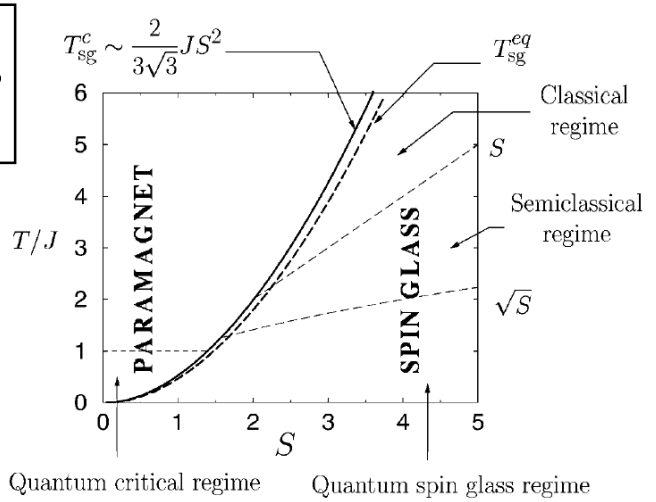
$\mu \propto L \Rightarrow \zeta \sim 1/2$

c.f. Arrachea & Rozenberg, PRL 86, 6172 (`01),
Oitmaa & Sushkov, PRL 87, 167206 (`01)

Heisenberg MFT

$$H = \frac{1}{\sqrt{n}} \sum_{i,j} J_{ij} \vec{S}_i \cdot \vec{S}_j \qquad \vec{S}_i \; SU(n) \; symmetry, \quad n \to \infty$$

[Georges, Parcollet, Sachdev `00]
[M. Vojta, Buragohain, Sachdev `00] (n=2)

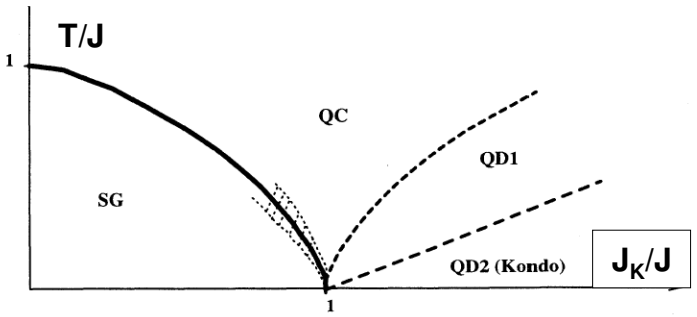


Metallic SG – MFT:

$$H_1 = \sum_{k,\alpha,i} \epsilon_k c^{(i)+}_{k\alpha} c^{(i)}_{k\alpha} + J_K \sum_i \vec{S}_i \cdot \vec{s}(i) - \sum_{i<j} J_{ij} S^z_i \cdot S^z_j$$

$$\vec{s}(i) \equiv \sum_{\alpha\beta} \sum_{k,k'} \frac{1}{2} c^{(i)+}_{k\alpha} \vec{\sigma}_{\alpha\beta} c^{(i)}_{k'\beta}$$

[Sengupta, Georges `95]



$$H = -\sum_{i<j,\alpha} t_{ij} c^{\dagger}_{i\alpha} c_{i\alpha} - \sum_{i<j,\mu} J^{\mu}_{ij} S_{i\mu} S_{j\mu} + H_{\rm int}$$

[Sachdev, Read, Oppermann `95]

Statics: QPT, Griffiths!
Dynamics?

Interactions between Spins / TLSs lead to low Temp. PT or (T=0) QPT and McCoy-Griffiths singularities.

But interactions lead also to dramatic (collective) dynamical effects:

Aging, Memory, Rejuvenation, FDT-violation, ...

Dynamics
Aging in Mean Field QSG

$$H_J[\Phi] = \frac{1}{2m}\sum_{i=1}^{N}\Pi_i^2 + \sum_{i_1,\ldots,i_p} J_{i_1,\ldots,i_p}\phi_{i_1}\cdots\phi_{i_p}\,, \qquad [\Pi_i,\phi_j] = -i\hbar\delta_{ij}$$

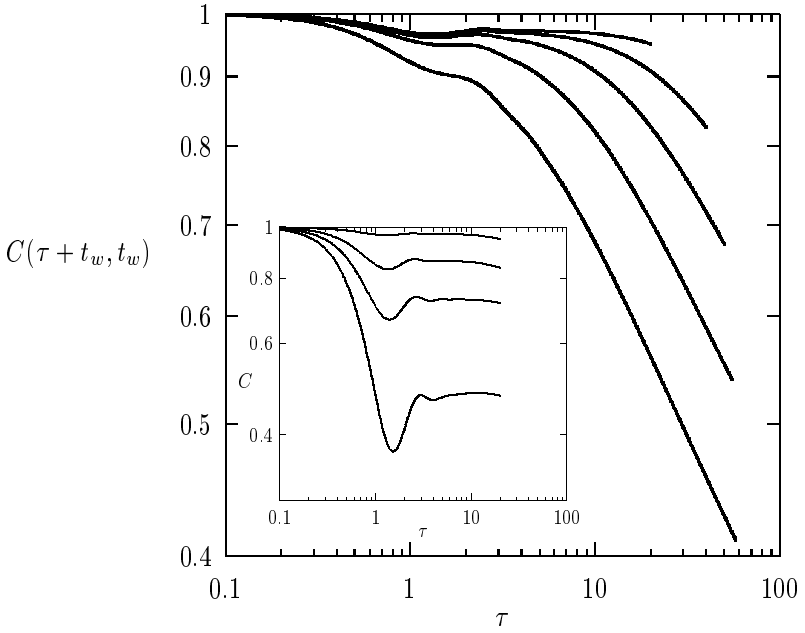
Ohmic bosonic bath with: $I(\omega) = (M\gamma_0/\pi)\,\omega\exp(-|\omega|/\Lambda)$

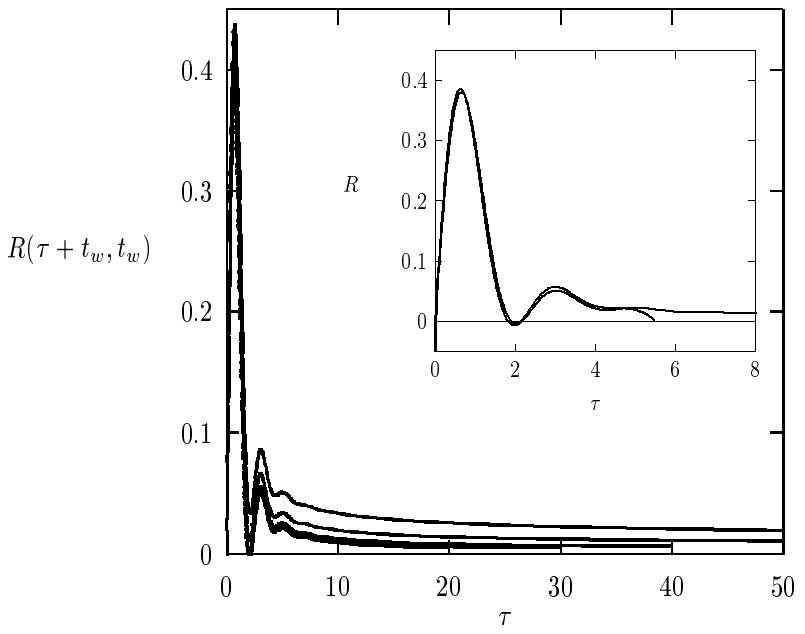
Quantum FDT: $R(\tau) = \frac{2i}{\hbar}\,\theta(\tau)\int \frac{d\omega}{2\pi}\,e^{-i\omega\tau}\tanh\left(\frac{\beta\hbar\omega}{2}\right)C(\omega)$

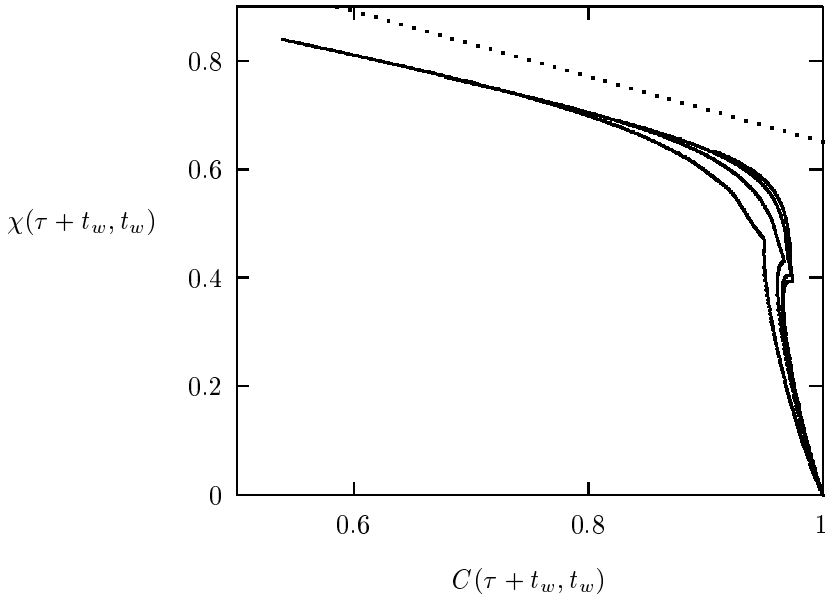
Out-of-Equilibrium QFDT:

$$R(t,t_w) = \frac{2i}{\hbar}\,\theta(t-t_w)\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\exp[-i\omega(t-t_w)]\tanh\left(\frac{X(t,t_w)\beta\hbar\omega}{2}\right)C(t,\omega)$$

Quantum FDT violation:







[Cugliandolo, Lozano `98]

Conclusions

- Random quantum spin chains (TIM, XX, XY, Heisenberg, XXZ):
  very good understanding (SDRG, analytical, QMC, DMRG).

- Infinite randomness fixed point (IRFP) –
  also in d>1 for Ising systems, not in Heisenberg AFM

- Griffiths-McCoy singularities:
  modify single spin behavior also away from the QCP.

- Short range QSG: Finite temperature phase transition in d>2;
  quantum phase transition in d>1.

- Heisenberg spin glasses: Dasgupta-Ma RG,
  d=2: $\omega=0$, $z=d$, $\zeta=1/2$; d=3: $\omega=0$, $z=d/2$, $\zeta=1/2$.

- Real time dynamics, aging, QFDT-violation in QSG with bosonic bath:
  only in mean-field yet.