An overview on non-relativistic and relativistic quantum Hall effects

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Motivation

Quantum Hall physics = study of 2D electrons in a strong magnetic field

- low-dimensional quantum physics
- electronic transport, beyond diffusive theory (mesoscopic physics)
  - conductance quantisation $G = \# \times e^2/h$
  - no microscopic conductivity $G \neq \sigma \times$ geometry
- strongly-correlated matter (incompressible liquids)
  - topological order
  - charge fractionalisation and fractional statistics
1. Introduction and Historical Overview
Classical Hall Effect (1879)

Quantum Hall system: 2D electrons in a \( B \)-field

Drude model (classical stationary equation):

\[
\frac{d\mathbf{p}}{dt} = -e \left( \mathbf{E} + \frac{\mathbf{p}}{m} \times \mathbf{B} \right) - \frac{\mathbf{p}}{\tau} = 0
\]
Shubnikov-de Haas Effect (1930)

(a) Hall resistance

(b) Density of states

oscillations in longitudinal resistance
→ Einstein relations $\sigma_0 \propto \frac{\partial n_{el}}{\partial \mu} \propto \rho(\epsilon_F)$
→ Landau quantisation (into levels $\epsilon_n$)

$$\sigma_0 \propto \rho(\epsilon_F) \propto \sum_n f(\epsilon_F - \epsilon_n)$$
Quantum Hall Effect (QHE)

\[ QHE = \text{plateau dans } R_H \& R_L = 0 \]

1980 : Integer quantum Hall effect (IQHE)
1982 : Fractional quantum Hall effect (FQHE)
**Integer Quantum Hall Effect**

Quantised Hall resistance at low temperatures

\[ R_H = \frac{h}{e^2} \frac{1}{n} \]

\( h/e^2 \): universal constant
\( n \): quantum number (topological invariant)

- result independent of geometric and microscopic details
- quantisation of high precision (\( > 10^9 \))

\[ R_{K-90} = 25,812,807 \, \Omega \]
system described by order parameter
(a) $\Delta_k = \langle \psi_{-k,\uparrow}^\dagger \psi_{k,\downarrow}^\dagger \rangle$ (superconductivity)
(b) $M^\mu(r) = \langle \psi_{\sigma}(r) \tau^\mu_{\sigma',\sigma} \psi_{\sigma'}(r) \rangle$ (ferromagnetism)

Ginzburg-Landau theory of second-order phase transitions (1957)

$$\Delta = 0 \quad \leftrightarrow \quad \Delta \neq 0$$

(disordered) \quad (ordered)

symmetry breaking
(a) broken $U(1)$ (gauge) symmetry
(b) broken $O(3)$ rotation symmetry

emergence of (collective) Goldstone modes
(a) superfluid mode, with $\omega \propto |k|$
(b) spin waves, with $\omega \propto |k|^2$
**Fractional Quantum Hall Effect**

partially filled Landau level → Coulomb interactions relevant

1983: Laughlin’s $N$-particle wave function

• no (local) order parameter associated with symmetry breaking
• no Goldstone modes
• quasi-particles with *fractional charges and statistics*

1990ies: description in terms of topological (Chern-Simons) field theories
Metal-Oxide Field-Effect Transistor (MOSFET)

usually silicon-based materials (Si/SiO₂ interfaces)
GaAs/AlGaAs Heterostructure

(a) AlGaAs
\[ E_F \]
\[ \bullet \bullet \bullet \bullet \bullet \]
dopants
GaAs

(b) AlGaAs
\[ E_F \]
\[ \bullet \bullet \bullet \bullet \bullet \]
dopants
GaAs

Reduced surface roughness (as compared to Si/SiO\(_2\))

⇒ enhanced mobility (FQHE)
Nobel Prize in Physics 2010: Graphene

Kostya Novoselov  Andre Geim
"for groundbreaking experiments regarding the two-dimensional material graphene"
What is Graphene?

Graphene = 2D carbon crystal (honeycomb)

- sp² hybridisation
- 2s
- 2pₓ
- 2pᵧ
- 2pᶻ

120°
Graphene and its Family

Graphene (2D)

Graphite

Nanotube (1D)

Footballène ($C_{60}$) (0D)
From Graphite to Graphene

(strong) covalent bonds in the planes

(weak) van der Waals bonds between the planes
How to Make Graphene: Recipe (1)

put thin graphite chip on scotch-tape
How to Make Graphene: Recipe (2)

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| fold scotch-tape on graphite chip and undo |
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~10
How to Make Graphene: Recipe (3)

glue (dirty) scotch-tape on substrate (SiO$_2$)
lift carefully scotch-tape from substrate
How to Make Graphene: Recipe (5)

place substrate under optical microscope
How to Make Graphene: Recipe (6)

zoom in region where there could be graphene
Electronic Mesurement of Graphene

Outline of Lectures

Mon 1: Introduction and Landau quantisation
Mon 2: Landau-level degeneracy and disorder/confined potential

Tue 1: Issues of the IQHE
Tue 2: Towards the FQHE, Laughlin’s wave function

Wed 1: Properties of Laughlin’s wave function: fractional charge and statistics, composite fermions
Wed 2: Chern-Simons theories, multi-component quantum Hall systems (1)

Thu 1: Multi-component quantum Hall systems (2)
Further Reading


2. Landau Quantisation and Integer Quantum Hall Effect
Bandstructure of Graphene
Infrared Transmission Spectroscopy

Grenoble high-field group: Sadowski et al., PRL 97, 266405 (2007)

selection rules:

\[ \lambda, n \rightarrow \lambda', n\pm1 \]
**Edge States**

LLs bended upwards at the edges (confinement potential)

chiral edge states ⇒ only forward scattering
Four-terminal Resistance Measurement

\[ R_L \sim \mu_3 - \mu_2 = 0 \]

\[ R_H \sim \mu_5 - \mu_3 = \mu_R - \mu_L \]

: hot spots

IQHE – One-Particle Localisation

(a)

density of states

\[ \varepsilon \]

\[ E_F \]

\[ n \]

\[ \text{NL} \]

\[ (n+1) \]

\[ R_{xx} \]

\[ R_{xy} \]

\[ \frac{h}{e^2 n} \]

\[ \nu = n \]

\[ B \]
IQHE – One-Particle Localisation

(a) $\varepsilon$ $E_F$ $n$ density of states $\rho_{NL}(n+1)$ $R_{xx}$ $R_{xy}$ $h/e^2 n$ $\nu = n$ $B$

(b) $\varepsilon$ $E_F$ $n$ density of states $\rho_{NL}(n+1)$ $R_{xx}$ $R_{xy}$ $B$
IQHE – One-Particle Localisation

(a) n

(b) (c) 

R_{xx}, R_{xy}

h/e^2n

v = n

B

R_{xx}, R_{xy}

h/e^2n

h/e^2(n+1)

B

R_{xx}, R_{xy}

B

localised states

extended states

density of states
**IQHE in Graphene**  


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**Density of states**

- **Graphene IQHE:**
  \[ R_H = \frac{h}{e^2} \nu \]
  at \( \nu = 2(2n+1) \)

- **Usual IQHE:**
  at \( \nu = 2n \)
  (no Zeeman)

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**Graphene IQHE Parameters:**

- \( V_g = 15 \text{V} \)
- \( T = 30 \text{mK} \)
- \( B = 9 \text{T} \)
- \( T = 1.6 \text{K} \)

\( \nu \sim \frac{1}{\nu} \)
Percolation Model – STS Measurement

2DEG on \( n \)-InSb surface Hashimoto et al., PRL 101, 256802 (2008)

(a)-(g) \( \frac{dI}{dV} \) for different values of sample potentials (lower spin branch of LL \( n = 0 \))

(i) calculated LDOS for a given disorder potential in LL \( n = 0 \)

(j) \( \frac{dI}{dV} \) in upper spin branch of LL \( n = 0 \)
Percolation Model – Scaling

scaling of the plateau width $\Delta B$


$\Rightarrow$ Second-order PT (QPT)

critical exponents: $1/z \nu = 0.42 \pm 0.04$ and $z \simeq 1$

$\Rightarrow \nu \simeq 2.3$

classical percolation: $\nu = 4/3$

special quantum model (numerics): $\nu = 2.5 \pm 0.5$
**Jain’s Wavefunctions (1989)**

*Idea:* “reinterpretation” of Laughlin’s wavefunction

\[
\psi^L_s(\{z_j\}) = \prod_{i<j} (z_i - z_j)^{2s}\chi_{p=1}(\{z_j\})
\]

\[\prod_{i<j} (z_i - z_j)^{2s}: \text{ “vortex” factor (2s flux quanta per vortex)}\]

\[\chi_{p=1}(\{z_j\}) = \prod_{i<j} (z_i - z_j): \text{ wavefunction at } \nu^* = 1\]
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\[\chi_{p=1}(\{z_j\}) = \prod_{i<j}(z_i - z_j):\]

wavefunction at \(\nu^* = 1\)

Generalisation to integer \(\nu^* = p\)

\[
\psi_{s,p}^J(\{z_j\}) = \mathcal{P}_{LLL} \prod_{i<j}(z_i - z_j)^{2s} \chi_p(\{z_j, \bar{z}_j\})
\]

\[\chi_p(\{z_j, \bar{z}_j\}): \text{wavefunction for } p \text{ completely filled levels}\]

\[\mathcal{P}_{LLL}: \text{projector on lowest LL (} \rightarrow \text{ analyticity)}\]
Physical Picture: Composite Fermions

CF = electron+”vortex” (carrying $2s$ flux quanta) with renormalised field coupling $eB \rightarrow (eB)^*$
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At $\nu = p/(2ps + 1) \leftrightarrow \nu^* = n_{el}/n_B^* = p$, with $n_B^* = (eB)^*/h$:

**FQHE of electrons = IQHE of CFs**
Generalisation: FQHE at half-filling

• 1987: Observation of a FQHE at $\nu = 5/2, 7/2$ (even denominator)

• 1991: Proposal of a Pfaffian wave function (Moore & Read; Greiter, Wilzcek & Wen)

$$\psi^{MR}(\{z_j\}) = \text{Pf} \left( \frac{1}{z_i - z_j} \right) \prod_{i<j} (z_i - z_j)^2$$

⇒ quasiparticle charge $e^* = e/4$ with non-Abelian statistics

• Further generalisations to $\nu = K/(K + 2)$: Read & Rezayi
**Multicomponent Systems**

- **A** physical spin: SU(2)
  
  (doubling of LLs)

- **B** bilayer: SU(2) isospin
  
  \[ \nu_+ = 1/2 \]
  
  \[ \nu_- = 1/2 \]
  
  \[ \nu_T = \nu_+ + \nu_- = 1 \]

- **C** graphene (2D graphite)
  
  two-fold valley degeneracy

  \[ \tau_1 \]
  
  \[ \tau_2 \]
  
  \[ \tau_3 \]

  \[ e_1 \]
  
  \[ e_2 \]
  
  \[ e_3 \]

  ○ : A sublattice
  
  ● : B sublattice

  spin + isospin : SU(4)