Quantum annealing and the random field Ising model

Mikko Alava

Helsinki University of Technology, Finland

Laboratory of Physics

Matti Sarjala, Vilho Petäjä (HUT)
Quantum and classical annealing

Classical energy landscape: use e.g. Simulated Annealing (lower the temperature).

Quantum version: modify a quantum parameter.
On Quantum Annealing

- Quantum annealing: system moves in the energy landscape by quantum tunneling.
- Small temperature, a finite quantum parameter $\Gamma$. Take $\Gamma$ or $\epsilon$ slowly towards zero ($H = H_0 + \epsilon H_1$).
- QM picture: induced (Landau-Zener) level crossings, follow lowest energy state.
Basic questions:

1. A quantum system $\rightarrow$ a classical one ($\Gamma = 0$).


3. how does this work vs. classical dynamics? In particular, if one wants to “optimize”.

4. this is what we study here!
Experimental case-study

\[ H = \sum_{ij} J_{ij} \sigma_i \sigma_j - \sum_i \Gamma \sigma_i^x \]

\(\sigma\) Pauli spin matrices.

\(LiHoF_4\):

tetragonal insulating FM.
Simple disordered Ising

(From cond-mat/0502....)

Residual energy:
\[ \varepsilon_{\text{res}} = \frac{(E_s - E_{gs})}{N}, \]
actual vs. groundstate energy.
We want to optimize!
Here, \( \varepsilon_{\text{res}} \) polynomial?
Satisfiability problem(s)

Santoro et al., cond-mat....
Apply quantum annealing to a NP-complete, K-SAT(isfiability) problem.
But: Classical Annealing better.
QA and a spin glass

Santoro et al. (Science 2002).
QA faster than CA
(simulated annealing).
Scaling theory: $\epsilon_{res} \sim \log N_{MC}^{-\zeta}$.
For QA: $2 \leq \zeta \leq 6$,
based on a Landau-Zener picture.
Random field Ising model

The Hamiltonian: \( H = -J \sum_{\langle i,j \rangle} \sigma_i^z \sigma_j^z - \sum_i h_i \sigma_i^z \)

\( h_i \) are the random fields at site \( i \), a (Gaussian) probability distribution \( P(h_i) \) w/ standard deviation \( \delta_h \).

The GS computed via the mapping to the max-flow/min-cut problem.
Why the RFIM?

• It is exactly solvable (and easily).

• It has a non-trivial energy landscape.

• RFIM: paramagnetic in 1D, 2D, phase transition in 3D. Overlap of GS with thermal state non-zero ($q$),

• Poorly known when quantum annealing is superior to classical. No real theory.
RFIM & QM

- QM effects can be tuned by varying a transverse field $\Gamma$.

The Hamiltonian reads:

$$
\mathcal{H} = J \sum_{\langle ij \rangle} \sigma_i^z \sigma_j^z - \sum_i h_i \sigma_i^z - \Gamma \sum_i \sigma_i^x,
$$

$\Gamma \sum_i \sigma_i^x$ stands for the quantum effects.

- QA: decrease $\Gamma$ from an initial value towards zero. $\Gamma(t)$? Usually, we use a logarithmical cooling schedule.

- $T > 0$: a $d$-dimensional quantum lattice Hamiltonian maps to a $d + 1$-dimensional classical (Suzuki-Trotter).
Suzuki-Trotter

\[ \mathcal{H}_{\text{eff}} = -\sum_{k=1}^{P} (J \sum_{\langle ij \rangle} \sigma_i^k \sigma_j^k - \sum_i h_i \sigma_i^k + J_\perp \sum_i \sigma_i^k \sigma_i^{k+1}) \]

\( P \) is the number of Trotter slices, \( J_\perp = -\frac{T_{\text{eff}}}{2} \ln(\tanh \frac{\Gamma}{T_{\text{eff}}}) \)

is the coupling between the spins in Trotter direction.

\( \mathcal{H}_{\text{eff}} \) simulated at the effective temperature \( T_{\text{eff}} = PT \).
Suzuki-Trotter II

An example of the mapping.
Typical run (1D)

Spin orientations vs. GS at 3 times
More details (1D)

An example of the decay of the magnetization. Recall: 1D paramagnetic.
Adding slices

More slices: better optimization. Recall that the size of the system becomes $P$-fold.
Effective temperature

Again, a smaller $T_{\text{eff}}$ results in a better $\epsilon_{\text{res}}$ but more slowly.
Paramagnetic phase

- Huse and Fisher [Phys. Rev. Lett. 57, 2205 (1986)]: arbitrary two-level systems in the case of simulated annealing, with an energy barrier between the states. Their result:

  \[ \epsilon_{res} \sim \log(N_{MC})^{-\zeta} \]

  where \( \zeta = 2 \).

- Santoro et al. [Science 295, 2427 (2002)] again: 2D SG (\( T = 0 \)-phase transition). \( \zeta > 2 \)?
General picture (1D)

What a surprise: more slices (P) and a smaller temperature work!

- The essential behavior in the paramagnetic phase.
- Idea: use quantum fluctuations ($P \sim \sqrt{N_{MC}}$).
- “things go wrong” - SA result recovered $\zeta = 2$.
- Things go well: $\zeta \rightarrow 6$?
2D case

In 2D, the RFIM “pseudo-FM” for $L$ small.
Thus, a cross-over expected.
3D considerations

• Now, a phase transition ($T(\delta h)$) - PM/FM.

• Huse and Fisher: in FM phase
  
  $\epsilon_{res} \sim \log(N_{MC})^{-\zeta}$

  with $\zeta = 1$.

• Nash et al. [Phys. Rev. B 43, 1272 (1991)]: activated scaling behavior observed, with $\zeta = 1.05 \pm 0.16$. 

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FM phase & Quantum annealing

- Also, a phase transition in 3D at some quantum parameter strength $\Gamma_c$.
- Using the energy barrier argument: the residual energy decreases as in classical case, but possibly with different exponent $\zeta_q$ [Phys. Rev. B 57, 8375 (1998)].
3D data

Note the parameters - and the SA result.
Final thoughts

• Cooling schedule (in)dependence in PM phases.

• PM: QA better. FM: not so.

• Where is the theory, $\zeta$?

• That is, no general knowledge how QA works for difficult problems (K-SAT, TSP etc.). Santoro...

• Use some kind of cluster algorithms?

• Bob Laughlin: “quantum computing is …”