Monte Carlo simulations of 2-d spin glasses

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Overview

• Model and motivation
• Simulation techniques
• Results for the ±J model
• Gaussian model, low energy excitations
• Conclusion and outlook
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Spin Glasses

Disordered magnetic system with random interactions: ex. $\text{Cu}_{1-x}\text{Mn}_x$ with $x \sim 1\%$

Interactions RKKY: $J(r) \sim \frac{\cos kr}{r^3}$

Frustration:

- Disorder + frustration => spin glass phase ($T_c \sim 15^\circ K$)
- Spins are frozen without apparent order
- Complex structure (many pure states ?)
- Rich dynamics (aging, memory)
Edwards-Anderson Model

\[ H_J(S) = -\sum_{\langle i,j \rangle} J_{ij} S_i S_j - B \sum_i S_i \]

Spins: \( S_i = \pm 1 \)

Random interactions:
\[ \langle J \rangle = 0 \quad \text{and} \quad \langle J^2 \rangle = 1 \]

typically \( J = \pm 1 \)

or \( P(J) \sim \exp(-J^2/2) \)
Overlaps

Overlap (=distance) between 2 configs $\alpha$ and $\beta$:  
\[ q^{\alpha\beta} = \frac{1}{N} \sum_i S_i^{\alpha} S_i^{\beta} \]

Link overlap:  
\[ q_{l}^{\alpha\beta} = \frac{1}{dN} \sum_{\langle i,j \rangle} S_i^{\alpha} S_j^{\alpha} S_i^{\beta} S_j^{\beta} \]

Volume $V$, surface $S$

\[ q = 1 - \frac{2V}{N} \]  
# different spins

\[ q_{l} = 1 - \frac{2S}{dN} \]  
# different bonds
Droplet theory \((d>2)\)

Only 2 pure states in the spin glass phase

Elementary excitations (droplets): compact with \(E \sim L^\theta\)

No spin glass phase in a magnetic field

\(P(q)\) is trivial
Mean field theory \((d=\infty)\)

Many pure states in the spin glass phase

Elementary excitations: system wide with \(E \sim 1\)

Spin glass phase under the A-T line in a magnetic field

\(P(q)\) is non-trivial
2-d Spin Glasses

• We can do it! Ground states, Monte Carlo.

• Critical temperature $T_c = 0$? (Yes)

• Universality?

• Is $d = 2$ the lower critical dimension?

• Behavior of $c$, $\xi$, $\chi$, $P(q)$... critical exponents?

• Does the droplet theory apply in 2 d?

• Nature of the low energy excitations (energy vs. size, fractal surface, scaling laws...).

• Does it say something for the 3-d case?
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Standard Monte Carlo

Metropolis: choose a spin at random and flip it with probability

\[ p = \min \left( 1, e^{-\beta \Delta E} \right) \]

Problem: very slow dynamics
Parallel tempering

Simulate $n$ temperatures in parallel with standard Monte Carlo

Exchange 2 configurations at different temperatures with probability

$$p = \min \left(1, e^{\Delta \beta \Delta E} \right)$$

Allow system to pass energy barriers
Cluster moves

Local overlap between 2 configurations $\alpha$ and $\beta$: $q_i = S_i^\alpha S_i^\beta$

Cluster: connected domain with constant $q_i$

Simulate 2 configurations in parallel

Choose a spin $i$ at random, find associated cluster and flip it in both configs (no rejection)

$q_i, q^{\alpha\beta}$ and $E = E^\alpha + E^\beta$ unchanged
Problems with cluster moves

When $d > 2$, $q_i$ defines only 2 clusters (percolation threshold $< 1/2$)

Flipping one cluster $= \text{Exchanging the configs}$

Even at $d = 2$, it does not equilibrate $q$ and $E$!

To equilibrate $q$: simulate more than 2 configs in parallel

Flipping a cluster between $\alpha$ and $\beta$ does not change $q^{\alpha\beta}$, but does change $q^{\alpha\gamma}$ and $q^{\beta\gamma}$

To equilibrate $E$: also use standard Monte Carlo and parallel tempering
Overview of the cluster algorithm

Simulate in parallel m configs for n temperatures with 3 moves:

Standard Monte Carlo for all m x n configs
Parallel tempering between the n temperatures
Cluster moves between m configs for each temperature
Efficiency

Orders of magnitude faster (here $\beta = 10, L = 100$)
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What we measure

Interactions ±J

Questions: \( T_c = 0 \) ? Correlation length \( \xi \) behavior ? Critical exponents

We measure:

- Spin glass susceptibility
  \[ \chi = N \langle q^2 \rangle \]
- Binder cumulant
  \[ g = \frac{1}{2} \left( 3 - \frac{\langle q^4 \rangle}{\langle q^2 \rangle} \right) \]

\( g \) is 0 is the paramagnetic phase and 1 in the spin glass phase

\( g \) is independent of \( L \) at \( T = T_c \) \( \Rightarrow \) the \( g \) curves intersect at \( T = T_c \)
$T_c = 0$
\[ \xi \sim (T - T_c)^{-\nu} \text{ ? No!} \]

\[ \chi \sim L^{2-\eta} \tilde{\chi}(TL^{1/\nu}) \]

\[ g \sim \tilde{g}(TL^{1/\nu}) \]
\[ \xi \sim e^{2\beta} \quad \text{Yes!} \]

\[ \chi \sim L^{2-\eta} \tilde{\chi}(2\beta - \ln L) \]

\[ g \sim \tilde{g}(2\beta - \ln L) \]
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Finding low energy excititations

We can compute the ground state

We have low temperature equilibrated configurations

Let’s compare them!

Each connected cluster boundary defines an elementary excitation
Using excitations as MC moves

During equilibration, build the list of all excitations under a given energy

Choose an excitation at random in the list and flip it with Metropolis probability

Efficiency increases as the temperature decreases!

Used with the cluster algorithm it allows to go to extremely low temperatures: $\beta = 50$ for $L = 100$

Does not work with $\pm J$ interactions: too many 0 energy excitations
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Conclusions and outlook

• Algorithms

  • A cluster algorithm for 2-d spin glasses
  • Use of ground state to find excitations
  • Use of excitations as Monte Carlo moves

• Results for $\pm J$ model

  • $T_c = 0$
  • $\xi \sim e^{2\beta}$

• Work in progress on the Gaussian model with A. Hartmann

  • Critical exponents
  • Low energy excitations
The End