Equilibrium and nonequilibrium properties of spin glasses in a field
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Outline

- Introduction to spin glasses (disordered magnets)
  - What are spin glasses?
  - Why are they interesting?
- Equilibrium properties of spin glasses in a field
  - Absence of an Almeida-Thouless line below upper critical dimension
- Nonequilibrium properties of spin glasses in a field
  - Return and complementary point memory effects
Introduction to spin glasses
Magnetic systems

- Prototype model for a magnet:
  \[ \mathcal{H} = - \sum_{\langle i,j \rangle} J_{ij} S_i S_j \quad S_i \in \{ \pm 1 \} \]

- Order parameter
  \[ m = \frac{1}{N} \sum_i S_i \]

- Disorder plays an integral role in nature:
  - Properties of materials change.
  - But often neglected.

Disorder
Spin glasses

- Phase transition into a glassy phase with no spatial order
- Complex energy landscape
- Slow dynamics
- Unexpected effects: aging, memory, hysteresis

Problem: Only mean-field model solvable. Solution: Simulations.

Numerically complex optimization problem, generally NP hard

Many applications to other fields and problems:
- Physics: vortex glasses, disordered magnetic media, error correcting codes, structural glasses, ...
- Computer science and related fields: pattern recognition, combinatorial optimization, economics, ...

Still a lot to be understood!
Brief history

- 1970: Canella & Mydosh see a cusp in $\chi_{SG}$ of Fe/Au alloys. The material has RKKY interactions

$$J_{ij} \sim \frac{\cos(2k_F R_{ij})}{R_{ij}^3}$$

which introduces disorder and frustration, necessary in a spin glass.

- 1975: Introduction of the Edwards-Anderson Ising spin glass model:

$$\mathcal{H} = -\sum_{\langle ij \rangle} J_{ij} S_i S_j$$

- 1975: The mean-field Sherrington-Kirkpatrick model is introduced.

- 1979: Parisi solution (RSB) of the mean-field model.

- 1986: Fisher & Huse suggest the droplet picture (DP) to describe short-range spin glasses.
Some open questions...

- Universality

- Memory effect

- Ultrametricity

- Spin-glass state in a field?
Equilibrium properties in a field
Spin-glass state in a field?

- Two contradicting predictions:
  - Replica Symmetry Breaking: Existence of an instability line [de Almeida & Thouless (78)] for mean-field glasses.
  - Droplet Picture: there is no spin-glass state in a field.

Which of the above pictures is correct?
What has been done?

- **Theory**: de Almeida & Thouless (78) predict an instability line for the SK model.

- **Experiments**:
  - Katori & Ito (94): claim existence of an AT line.
  - Mattson et al. (95): no AT line (study divergent relaxation times).

- **Simulations**:
  - Study of the Binder cumulant [Bhatt & Young (85), Kawashima & Young (96)]: no AT line. Problem: Binder ratio not stable in a field.
  - Out of equilibrium methods [Marinari et al. (98)]: signature of an AT line. Problem: Is the true equilibrium behavior probed?
  - Simulations according to experimental protocols [Takayama & Hukushima (04)] show no AT line.
  - Zero-T calculations [Houdayer & Martin (99), Krzakala et al. (02)] find not AT line.

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Katzgraber & Young, PRL 93, 207203 (2004)
First approach: 3D EA Ising spin-glass

- Edwards-Anderson Ising spin-glass model with random fields:

\[ \mathcal{H} = -\sum_{\langle ij \rangle} J_{ij} S_i S_j - \sum_i h_i S_i \]

- Properties:
  - Sum over nearest neighbors in 3D with Gaussian random bonds.
  - The random fields are Gaussian distributed with zero mean and 
    \[ [h_i^2]_{av}^{1/2} = H_R \]. This corresponds to a uniform field \( H_R \).
  - For zero field \( T_c \approx 0.95 \).
  - Why do we choose random fields?
    - Equilibration test for the Monte Carlo method
    - Parallel tempering performs slightly better than in a uniform field.
Parallel tempering Monte Carlo

• Simulate $M$ copies of the system at different temperatures with $T_{\text{max}} \gg T_c$.

• Allow swapping of neighboring temperatures: easy crossing barriers!

• Fast equilibration with rough energy landscapes.

• The method obeys detailed balance

$$P(S_{m+1} \leftrightarrow S_m; \beta_{m+1} \leftrightarrow \beta_m) = \begin{cases} 
  e^{-\Delta} & : \Delta > 0 \\
  1 & : \Delta \leq 0
\end{cases}$$

$$\Delta = (\beta_{m+1} - \beta_m)(E_m - E_{m+1})$$
Ballesteros et al. (00) reintroduce the use of the finite-size correlation length to study phase transitions in spin glasses.

Calculation of $\xi_L$:

- Wave-vector-dependent connected spin-glass susceptibility:
  \[ \chi_{SG}(k) = \frac{1}{N} \sum_{i,j} \left[ \left( \langle S_i S_j \rangle_T - \langle S_i \rangle_T \langle S_j \rangle_T \right)^2 \right] e^{i k \cdot (R_i - R_j)} \]

- Ornstein-Zernicke approximation:
  \[ [\chi_{SG}(k)/\chi_{SG}(0)]^{-1} = 1 + \frac{\xi_L^2 k^2}{2} + \mathcal{O}[(\xi_L k)^4] \]

- Compensate for PBC and finite-size effects and solve for $\xi_L$
  \[ \xi_L = \frac{1}{2 \sin(k_{min}/2)} \left[ \frac{\chi_{SG}(0)}{\chi_{SG}(k_{min})} - 1 \right]^{1/2} \]

- Finite-size scaling:
  \[ \frac{\xi_L}{L} = \tilde{X} \left( L^{1/\nu} [T - T_c(H_r)] \right) \]
How well does this work for $H = 0$?

- The data cross at $T_c \approx 0.95$ in agreement with previous results.
- Evidence of a spin-glass state for $T \leq 0.95$. 

![Graph showing $\xi_L/L$ vs. $T$ for different $L$ values, with $H_r = 0$.]
Finite fields... No transition

- Using parallel tempering we can scan down to $T = 0.23$.
- We perform slices at different fields.
- Krzakala predicts $H_{AT} \approx 0.65$.

\[ H_{AT} \approx 0.65 \]

\[ T_{c} \approx 0.95 \]

\[ H_{MF} \approx 2.10 \]

\[ H_{AT} = 0.50 \]

\[ T_{min} = 0.23 \]

\[ H_{min} = 0.05 \]
Finite fields... No transition

- Using parallel tempering we can scan down to $T = 0.23$.
- We perform slices at different fields.

\[ H \approx 0.30 \]
Finite fields... No transition

- Using parallel tempering we can scan down to $T = 0.23$.
- We perform slices at different fields.

\[
\begin{align*}
T &= 0.23 \\
H &= 0.10
\end{align*}
\]
Finite fields... No transition

Problem: small systems.
- Using parallel tempering we can scan down to $T = 0.23$.
- We perform slices at different fields.

Maybe AT line for $d > 6$?

Does the method pick up the AT line?

No AT line in 3D.

What about higher dimensions?

Problem: small systems.
Solution: 1D chain

\[ \mathcal{H} = -\sum_{ij} J_{ij} S_i S_j - \sum_i h_i S_i \]

- The sum ranges over all spins.
- Gaussian random fields and power-law modulated random bonds (SK model for \( \sigma = 0 \)):
  \[ [h_i^2]^{1/2} = H_r \]
  \[ J_{ij} \sim \frac{\epsilon_{ij}}{r_{ij}^\sigma} \]

The model allows for a large range of sizes.

Interesting regime: \( 1/2 < \sigma < 1 \)
1D chain: zero field

- The data span a large range of sizes
- Transition in zero field for $\sigma < 1.0$
- $T_c \approx 1.00$

$\sigma = 0.55$
1D chain: zero field

- The data span a large range of sizes
- Transition in zero field for $\sigma < 1.0$
- $T_c \approx 0.85$

$$
\sigma \approx 0.65
$$
The data span a large range of sizes.

Transition in zero field for \( \sigma < 1.0 \).

\[ T_c \approx 0.68 \]
1D chain: zero field

- The data span a large range of sizes
- Transition in zero field for $\sigma < 1.0$
- $T_c \approx 0.48$

$$
\sigma = 0.85
$$

$$
T_c = 0 \quad T_c > 0
$$

$$
\sigma_d = \frac{SK_l \sigma_c(d)}{d = 2 \sigma_{SR} LR SR_d \frac{1}{2} \sigma^2 + IR MF LR}
$$

$$
\sigma < 1.0
$$

$$
T_c \approx 0.48
$$
1D chain: finite field ($H = 0.10$)

- The data span a large range of sizes
- Mean-field behavior for $\sigma \leq 2/3$
- $T_c \approx 0.92$

\[
\sigma_d = \frac{\text{SK}}{\text{lr}} \sigma_c(d) = \frac{\sigma_2}{2^{1/2}} \sigma_0 + \text{IR MF LR SR}
\]
1D chain: finite field ($H = 0.10$)

- The data span a large range of sizes
- Mean-field behavior for $\sigma \leq 2/3$
- Crossover regime

![Graph showing phase transitions and critical temperatures]

$T_c = 0$

$\sigma = 0.65$
1D chain: finite field \((H = 0.10)\)

- The data span a large range of sizes
- Mean-field behavior for \(\sigma \leq 2/3\)
- \(T_c = 0\)
The data span a large range of sizes

Mean-field behavior for $\sigma \leq 2/3$

$T_c = 0$
What have we learned so far?

- The AT line vanishes when not in the mean-field regime.
- For short-range spin glasses below the upper critical dimension:
  - Related work:
    - Proposal by M.A. Moore (cond-mat/0508087) how RSB might be stable for $d < 6$ (Temesvari: RSB for $d > 8$).
    - Proposal by de Dominicis (cond-mat/0509096) of a possible field theory for DP for $d < 6$.
    - See also: http://jc-cond-mat.bell-labs.com/jc-cond-mat/
Nonequilibrium properties in a field
Hysteresis in disordered spin systems?

- Due to the randomness the system has a rough energy landscape.
- The rough energy landscape has many metastable states responsible for the hysteresis.

\[ H \geq H_{\text{crit}} \]

\[ M - H \]

\[ E_0 - E \]
RPM & CPM

- **Definitions:**
  - **Complementary point memory:**
    Correlations between configurations at $+H^*$ and $-H^*$.
  - **Return point memory:**
    Configurations at a given $H^*$ are similar after $n$ loop cycles.
- **Example:** Barkhausen noise.
- **Recent experiments** [Pierce et al. (05)] and numerical work [Deutsch & Mai (05), Jagla (05)] suggest the following:
  - RPM and CPM $\rightarrow 0$ for decreasing disorder.
  - CPM $<$ RPM $<$ 1 for systems with high disorder.

Can we see these effects in simple models?
Previous results

• Experiments by Pierce et al. (05):

Measure the effects of disorder on Co/Pt multilayer films using X-ray speckle metrology.

• Simulations by Deutsch & Mai (05) [Jagla (05)] using LLG dynamics:

\[ \mathcal{H} = -J \sum_{<i,j>} S_i S_j - \alpha \sum_i (S_i n_i)^2 - w \sum_{i \neq j} \frac{1}{r_{ij}^3} [3(S_i \cdot e_{ij})(S_j \cdot e_{ij}) - S_i S_j] - H \sum_i S_i^z \]

• Results of theory and simulation agree.
Models studied here

- Edwards-Anderson Ising spin glass (EASG):
  \[ \mathcal{H} = - \sum_{\langle ij \rangle} J_{ij} S_i S_j - H(t) \sum_i S_i \quad S_i \in \{\pm 1\} \]
  - Gaussian-distributed bonds: \([J_{ij}]_{av} = 0\) and \([J_{ij}^2]_{av}^{1/2} = \sigma_J\)
  - Nearest neighbor interactions in two dimensions.
  - From spin reversal symmetry expect: \(\text{RPM} = \text{CPM} = 1\) for \(T = 0\).

- Random-field Ising model (RFIM):
  \[ \mathcal{H} = -J \sum_{\langle ij \rangle} S_i S_j - \sum_i h_i S_i - H(t) \sum_i S_i \]
  - Gaussian-distributed random fields \([h_i]_{av} = 0\) and \([h_i^2]_{av}^{1/2} = \sigma_h\)
  - Nearest neighbor interactions in two dimensions.
  - No spin reversal symmetry: \(\text{CPM} < 1\)
  - Due to the no passing property we expect \(\text{RPM} = 1\) for \(T = 0\).
Algorithm

• T = 0
  • Change the external field in small steps. Compute the local fields:

  \[ h_i = \sum_j J_{ij} S_j + H(t) \]

  of each spin \( S_i \). A spin is unstable if \( h_i S_i < 0 \). Dynamics:
  • Flip a randomly chosen unstable spin.
  • Update the local fields of the neighbors.
  • Iterate until all spins are stable.

• T > 0
  • Change the external field in small steps.
  • For each field step perform a finite-T Monte Carlo simulation.
  • Iterate until the magnetization is independent.

Average over 500 disorder realizations.
EASG: Qualitative behavior

- Intermediate disorder:
  \[ \sigma_J = 1 \]
- \( T = 0.2 \)
- Red pixels denote differences between configurations.
- RPM and CPM are not perfect due to frustration.

Can we better quantify this?
Overlaps to measure RPM & CPM

- Idea:
  - Study correlations between configurations.
  - Start the loop at positive saturation.
  - Definition of overlaps: $q$ measures the degree of memory configurations, $q'$ the uniqueness.

- CPM:
  \[
  q(H^*) = - \frac{1}{N} \sum_{i=1}^{N} S_i(H_{I^*}) S_i(-H_{II^*}) \quad q'(H^*) = - \frac{1}{N} \sum_{i=1}^{N} S_i(H_{I^*}) S_i(H_{II})
  \]

- RPM:
  \[
  q(H^*) = \frac{1}{N} \sum_{i=1}^{N} S_i(H_{I^*}) S_i(H_{I_{I'}}^*) \quad q'(H^*) = \frac{1}{N} \sum_{i=1}^{N} S_i(H_{I^*}) S_i(H_{I'})
  \]
EASG: Overlap \( q(H^*) \)

\[
q(H^*) = -\frac{1}{N} \sum_{i=1}^{N} S_i(H^*_I)S_i(-H^*_II)
\]

- Data show strong correlations between configurations.
- Memory not perfect even at \( T = 0 \).
- Memory decreases with \( T \).

\[
\sigma_J = 1
\]

\( T = 0.0 \)
\( T = 0.2 \)
\( T = 1.0 \)
EASG: Overlap $q'(H)$

$$q'(H^*) = -\frac{1}{N} \sum_{i=1}^{N} S_i(H^*_1)S_i(H^*_2)$$

- Correlations unique.
- Memory not perfect even at $T = 0$.
- Memory decreases with $T$.
- RPM = CPM.

$\sigma_J = 1$
EASG: Overlap $q(H^*)$ disorder scan

$$q(H^*) = -\frac{1}{N} \sum_{i=1}^{N} S_i(H^*_I) S_i(-H^*_I)$$

- Data for $T = 0.2$.
- Memory better for increasing disorder.
- Variable $\sigma_J$
- Qualitative agreement with the experiments.
RFIM: Overlap $q(H^*)$

$$q(H^*)_{CPM} = -\frac{1}{N} \sum_{i=1}^{N} S_i(H^*_I) S_i(-H^*_I)$$
$$q(H^*)_{RPM} = \frac{1}{N} \sum_{i=1}^{N} S_i(H^*_I) S_i(H^*_I')$$

- CPM < RPM.
- RPM = 1 (T = 0).
- CPM < 1 (T = 0).
- $q \rightarrow -1$?
- $\sigma_h = 1$
RFIM: Overlap $q(H^*)$

\[ q(H^*) = -\frac{1}{N} \sum_{i=1}^{N} S_i(H^*_I)S_i(-H^*_II) \]

- Variable $\sigma_h$
- Data for $T = 0.20$
- CPM $\sim 0$
- Anticorrelations for large disorder (loops close).
RFIM: Overlap $q(H^*)$

$q(H^*) = \frac{1}{N} \sum_{i=1}^{N} S_i(H^*_I) S_i(H^*_I')$

- Variable $\sigma_h$
- Data for $T = 0.20$
- Memory better for higher disorder
- Narrow dips due to sharp loops
Summary & comparison

- Experiments of Pierce et al.:
  - CPM < RPM < 1 for all T
  - No error bars: CPM < RPM?
  - RPM and CPM increase with disorder.

- Edwards-Anderson Ising spin glass:
  - Model has frustration and spin reversal symmetry.
  - RPM = CPM for all T, RPM and CPM < 1 (also T = 0).
  - RPM and CPM increase for increasing disorder.

- Random-field Ising model:
  - No frustration and no spin reversal symmetry.
  - RPM > CPM for all T, RPM = 1 for T = 0, CPM ~ 0 for all T.
  - RPM increases for increasing disorder.

No perfect agreement.

Can we construct a minimal model which reproduces the experiments?
\[ H_{SG+RF} = -\sum_{\langle i,j \rangle} J_{ij} S_i S_j - \sum_i [H + h_i] S_i \]

- 5% random fields with \( \sigma_h = 1 \)
- variable \( \sigma_J \)
- CPM < RPM < 1
- Memory increases with increasing disorder
- The random fields break spin-reversal symmetry
- Deutsch: break time-reversal symmetry.
Why does the memory increase with increasing disorder?

- Strong disorder $\rightarrow$ Rough energy landscape.
- Rough energy landscape $\rightarrow$ Pinning in configuration space.
- Pinning in Configuration space $\rightarrow$ Increased memory.

Analogy: Colorado River vs Nile Basin.
Concluding remarks

- **Equilibrium properties:**
  - Simulations on the one-dimensional Ising chain suggest that short-range spin glasses can have an AT line for $d > 6$.

- **Nonequilibrium Properties:**
  - The random-field Ising model and the EA spin glass show memory effects.
  - The SG+RF model is a minimal model which shows the same behavior as the experiments of Pierce et al.

- **Future problems:**
  - Probe other characteristics of the mean-field model on short-range systems, such as ultrametricity.
  - Understand the nature of the spin-glass state (RSB favored in zero field, DP favored in a finite field).
Thank you.