GEORG-AUGUST-UNIVERSITÄT GÖTTINGEN

Fakultät für Physik

Master's Thesis

Supergravity & Supernovae – Gravitino Phenomenology in Astrophysics

prepared by

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Thesis period:	21st October 2012 until 11th November 2013
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Abstract

In supergravity models with superlight gravitinos $\tilde{G}$ the gravitino couplings to matter are amplified by the tiny mass of this particle. Such models provide additional supernova cooling mechanisms via light gravitino emission. These states would appear to the observer as missing energy since gravitinos cannot be detected. The detection of supernova neutrinos from SN1987A however provides us with a clear bound on any extra cooling mechanism of supernova cores. Any suggested new physics like supergravity has to respect these bounds and we derive the lower bounds on the gravitino mass obtained from this requirement.
We investigate the two gravitino pair production processes $\gamma\gamma \longrightarrow \tilde{G}\tilde{G}$ and $\nu\overline{\nu} \longrightarrow \tilde{G}\tilde{G}$ and derive corresponding bounds that strongly depend on the masses of the goldstino's scalar partners. We also discuss phenomenological implications of bilinear R-parity violations and show that the corresponding additional production of single gravitinos from the processes $\gamma\gamma \longrightarrow \tilde{G}\nu$ and $\nu\overline{\nu} \longrightarrow \tilde{G}\nu$ is too low to have any observable effect.

Keywords: Supergravity, Astroparticle Physics, Gravitino Phenomenology, Supernovae

Zusammenfassung

Die Masse des Gravitinos $\tilde{G}$ kann in einigen Modellen der Supergravitation sehr gering sein. Die Materiekopplungen eines leichten Gravitinos werden durch den kleinen Wert der Masse $m_{3/2}$ verstärkt. Im Kern einer Supernova wären so die Energien hoch genug, Gravitinos zu produzieren, zu emittieren und den Supernovakern zusätzlich abzukühlen. Durch die Detektion der Neutrinos der Supernova SN1987A wurde jedoch verifiziert, dass der größte Energieanteil durch Neutrinos abgestrahlt wird, neue relevante Supernova-Kühlungsmechanismen sind somit ausgeschlossen und neue Physik darf solche nicht vorhersagen.
Wir untersuchen die Produktion von leichten Gravitino-Paaren durch die Prozesse $\gamma\gamma \longrightarrow \tilde{G}\tilde{G}$ und $\nu\overline{\nu} \longrightarrow \tilde{G}\tilde{G}$ und finden untere Grenzen für $m_{3/2}$, da leichtere Gravitinos zu stark zu der Energieemission beitragen würden. Diese Grenzwerte hängen jedoch stark von der Annahme ab, wie massiv die Superpartner des Goldstinos in dem gewählten Modell sind. Des weiteren diskutieren wir, inwiefern die Hinzunahme von bilinearen R-Paritätsverletzungen unsere Ergebnisse beeinflussen. Die neuen Produktionskanäle einzelner Gravitinos $\gamma\gamma \longrightarrow \tilde{G}\nu$ und $\nu\overline{\nu} \longrightarrow \tilde{G}\nu$ erweisen sich jedoch als zu schwach, um bei der Kühlung einer Supernova eine relevante Rolle zu spielen.

Stichwörter: Supergravitation, Astroteilchen-Physik, Gravitino Phänomenologie, Supernovae

Contents

1
Preface

Almost four decades passed by since the Standard Model of Particle Physics has been formulated. It summarizes today's best understanding of the elementary particles of nature and their fundamental interactions. For these forty years the Standard Model has been scrutinized like hardly any other theory. Countless high-energy experiments have been conducted in hopes that new phenomena would be observed; Phenomena that could not be explained by the Standard Model and whose discovery would lead to scientific progress extending our fundamental understanding of Nature. Over the years the experimental efforts became bigger and bigger and the discoveries confirmed exactly what the Standard Model predicted. This year the Nobel prize in physics was awarded to the British physicist Peter Higgs and the Belgian physicist François Englert for their theoretical discovery of the Higgs-mechanism. The reason for this was the discovery of a very Higgs-like boson at CERN a year before. It seems that the Standard Model as a whole has been comfirmed as the correct description of high-energy physics at accessible energies. The Large Hadron Collider at CERN will presumably return to operation

in 2015 and the hopes to find new physics are still high, but one might ask how to proceed if the LHC fails to detect any physics beyond the Standard Model.
Nevertheless, it is undisputed that the Standard Model is not able to serve as a final fundamental theory but as an effective description of high-energy physics. We will depict its problems in more detail in sec. 2.1. There is a vast number of proposals on how to extend the theory and it is the purpose of particle physics phenomenology to find ways to examine and check the new ideas using empirical observations. Particle colliders are only one experimental setting with high enough energies, another way is to exploit the natural high-energy phenomena in astrophysics, even if these energies are lower than in today's colliders.
In this thesis we make use of the observation of the famous Supernova SN1987A. As we will see, the detection of Supernova neutrinos enables us to test any new physics that would lead to novel mechanisms of energy loss in such an event. One of such mechanisms could be provided by Supersymmetry.
Supersymmetry is a special extension of the Standard Model's symmetry group that gained a lot of attention during the last decades. In the course of this thesis we depict what Supersymmetry is and show its ability to solve several of the Standard Model's most severe problems. Yet, it also predicts many new phenomena, e.g. every particle from the Standard Model would obtain a partner particle, called the superpartner or 'sparticle'. None of these new particles has been observed so far.
By promoting Supersymmetry to a local symmetry gravity is included into this framework very naturally. In Supergravity, the superpartner of the graviton is called the gravitino or $\tilde{G}$. Different realizations of supergravity, i.e. different ways of SUSY breaking, can give rise to both heavy and very light gravitinos. The latter case is of special interest for us, because a small gravitino mass could lead to additional contributions of Supernova cooling beside neutrino emission. Since these new mechanisms are strictly bound by the SN1987A observation we can derive certain bounds on the gravitino mass. This is the main goal of this thesis.

We organize the way of proceeding as follows. In ch. 2 we start by discussing the context of this thesis. We give some more details on the Standard Model and the hierarchy problem and devote one section to Supernovae and the energy-loss argument. Subsequently we introduce the idea of Supersymmetry and Supergravity in ch. 3. We cover topics like Supersymmetry breaking, the Super-Higgs effect, R-Parity and present the full general Supergravity Lagrangian. Afterwards we focus on the gravitino and depict its phenomenology in ch. 4, which is completed by the Feynman rules which we obtain from the Lagrangian in ch. 3.

Our main analysis is divided into two parts. In ch. 5 we review some investigations by Grifols, Mohaparta and Riotto [1] in great detail and generalize them. In ch. 6 we examine how these results are altered in the case of broken R-parity. We summarize and discuss our results in the last chapter.

Since we want to present our analysis as comprehensibly as possible, we provide several appendices containing many helpful treatments. We establish the conventions, notations and physical constants used throughout this thesis in app. A. We devote an extra appendix (app. B) to spinor notations and conventions and also treat spinors in curved spacetime. This is necessary for app. C where we derive Feynman rules for gravitons from linearized gravity. The appendices D, E and F contain several other relevant relations we need and tools we use.

2

Fundamentals – From Particles to Stellar Explosions

This thesis can be placed into the broad spectrum of astroparticle physics. We start by discussing the particle related background as well as the astrophysical context.

Supersymmetry (and Supergravity) is a well-motivated extension of the Standard Model. As an introduction we treat the general properties and problems of the SM. Here we focus on the hierarchy problem that serves as a nice transition and motivation to the topic of Supersymmetry.

In sec. 2.2 we treat the astrophysical setting of this thesis and discuss the mechanism of supernovae. We are most interested in the energy-loss-argument which is the basis for our subsequent investigations.

2.1. The Standard Model of Particle Physics – Its Successes and Deficits

The Standard Model of Particle Physics (SM)[1] is an extremely successful description of fundamental particles and their interactions. Despite gigantic efforts by thousands of physicists it was not possible to observe significant anomalies from the SM predictions. The most recent success of the SM is the confirmation of the predicted existence of the (or rather a) Higgs boson on 4 July 2012 [5] at the Large Hadron Collider (LHC) at the European Organization for Nuclear Research (CERN) in Geneva. Assuming that the observed boson is indeed the SM-Higgs one finds a remarkable agreement between experiment and theory [6]. However we will see that the SM is in a somewhat awkward situation from the theoretical as well as from the empirical point of view, despite its large achievements.
The SM is a Poincaré invariant quantum field theory with additional internal symmetries. The corresponding symmetry groups as well as the field content and the values of the 19 free parameters of the theory are dictated by observations. Having non-abelian gauge groups the SM is formulated as a Yang-Mills-Shaw theory introduced in 1954 by Chen-Ning Yang and Robert L. Mills and independently by Ronald Shaw in 1955 [7]. The structure of the SM interactions are determined by the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$. It is regarded as one of the most successful theories in science. Yet, the SM faces some serious problems or challenges if you will. We want to mention three of these challenges, two of which are concerned with the SM's incompleteness and one that concerns its internal consistency.
The SM cannot be the fundamental and complete description of nature since it does not incorporate all of either nature's interactions or nature's particles. The issue of gravity is not addressed in the SM, it just does not appear. A consistent formulation of a quantum theory of gravity and its relation to particle physics remains one of the most important open questions in physics today.
The second problem of incompleteness is the lack of Dark Matter (DM) in the SM. On the scale of galaxies, galaxy clusters and cosmology, we find convincing

[1] For a pedagogical introduction we refer e.g. to the textbook of Halzen and Martin [2] or the review by Novaes [3]. For more details of the historical development of the SM we mention [4] as an example.

evidence for the existence of huge quantities of invisible and massive matter [8]. Models in which the DM particle is a Weakly Interacting Massive Particle (WIMP) are favored from cosmological arguments, yet none of the particles in the SM have the right properties in order to act as DM. The last challenge is one regarding the internal consistency of the SM . It is called the 'hierarchy problem' [9] and it will serve us as a bridge to the topic of this thesis[2].

Not least because of the shortcomings of the SM mentioned above, today's interpretation of the SM is that of an effective field theory. The theory is not interpreted as a fundamental theory but as a description of physics up to a physical cutoff energy Λ_{UV}. Above this scale new, unknown physical structures become relevant, e.g. quantum gravity effects may no longer be ignored at the Planck scale.

Considering this new physics it is puzzling that the Higgs mass is as small as it is. A fundamental scalar field as the Higgs field is highly sensitive to the heaviest fields in a theory and any physics beyond the SM. The bare mass would receive radiative corrections,

$$m_H = m_{H,\text{bare}} + \Delta m_H = \mathcal{O}(100\text{GeV})\,. \tag{2.1}$$

If the Higgs couples e.g. to some heavy fermion f via the term $\lambda_f H\overline{f}f$ you obtain a mass correction from the diagram in figure 2.1.

This diagram gives corrections to the Higgs mass of the form

$$\Delta m_H^2 = -\frac{|\lambda_f|^2}{8\pi^2}\Lambda_{\text{UV}}^2 + \dots\,. \tag{2.2}$$

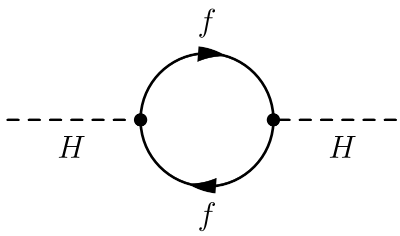


Figure 2.1.: Fermion Loop

Since we are sure of the existence of physics beyond the SM (at least gravity), we expect Λ_{UV} to be a large energy scale like the GUT or Planck scale, yielding large correction terms like (2.2). Therefore a cancellation between two huge terms must occur in (2.1) in order to get a small value of $m_H \approx 125$ GeV.

In anticipation of the next chapters we mention that this problem could be solved

[2] Here we follow mainly the introduction of [10]

by the introduction of a new symmetry connecting bosons and fermions. If we assume that for each fermion in the theory we also have a bosonic field, we obtain additional corrections to the Higgs mass, e.g. given by the diagram in figure 2.2,

$$\Delta m_H^2 = \frac{\lambda_s}{16\pi^2}\Lambda_{\mathrm{UV}}^2 + \dots . \tag{2.3}$$

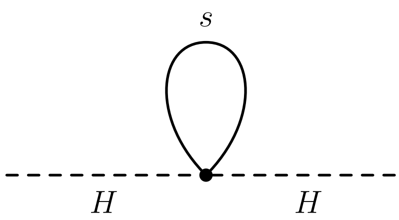


Figure 2.2.: Scalar Loop

The different sign between (2.2) and (2.3) enables us to understand the cancellation of radiative corrections without the need of fine-tuning [11]. In chapter 3.1 we will elaborate on this kind of symmetry.

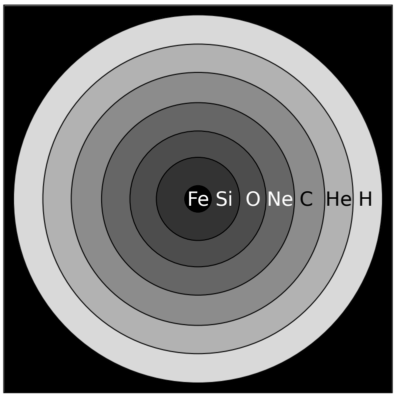
We observe that recent developments at the LHC have left the SM in an inconvenient position. On the one hand we have this extremely powerful and predictive theory, on the other hand, for the reason mentioned above, it cries out for extensions and particle physicists around the world hoped for the LHC to not only find the Higgs boson but also evidence of new physics at the TeV scale. However the first run of the LHC, that ended on 14 February 2013, seems to confirm the SM and fails to detect signals from physics Beyond the Standard Model (BSM) [12]. The situation will worsen if the second run of the LHC at a center-of-mass energy of 13 TeV, beginning presumably in early 2015[3], does not show anything new as well.

This would leave us with the serious question of how to proceed in high energy physics. Of course we have to wait for the results, but in either case it is important to look for alternative methods to test high-energy physics beyond accelerators. Having a vast number of proposed extensions and hypotheses and limited amounts of data finding new phenomenological methods has to be the order of the day. As mentioned in the preface one promising area of research is astroparticle physics. The relatively young combination of particle physics, astrophysics and cosmology faces some of the most fundamental open questions in modern physics and reveals many new possibilities to tackle these problems. This thesis gives one of these attempts.

[3] `http://press.web.cern.ch/press-releases/2012/12/`

2.2. Supernovae and the Energy Loss Argument

A core collapse of stars with masses larger than a few solar masses can trigger a giant stellar explosion called a Supernova (SN) [13]. Its dynamics can be explained by a 'bounce-and-shock' model [14]:
The evolution of a star ends as soon as its nuclear fuel is exhausted. Then the central iron core is surrounded by various layers of different fusion processes, e.g. hydrogen fusion in the most outer layer as illustrated in the figure.



The iron core can no longer release energy via fusion and remains stable as long as the electron degeneracy pressure balances the gravitational pressure. But once the core mass exceeds the Chandrasekhar limit of $M \approx 1.44 M_\odot$ it becomes unstable and starts to collapse. This contraction enhances itself by leading to electron-capture ($p + e^- \to \nu_e + n$) and lowering the electron pressure. In a split second the core collapses to a small ($R_{SN} \approx 10$km) and hot ($T_{SN} \approx 50$MeV) object of supranuclear density ($\rho_{SN} \approx 3 \times 10^{14}\,\mathrm{g\,cm^{-3}}$). Then the contraction slows down and the gravitational binding energy of

$$E_b \sim \frac{3}{5}\frac{G_N M}{R_{SN}} = 1.6 \times 10^{53} \left(\frac{M}{M_\odot}\right)\left(\frac{R_{SN}}{10\mathrm{km}}\right)^{-1} \mathrm{erg} \tag{2.4}$$

gets released. While the contraction of the core stops more material continues to fall towards the core's center with suprasonic velocity. This leads to the emergence of a shock wave at the core edge. This shock wave moves outwards and gains more and more energy. Finally it ejects the material of the stellar mantle. This explosion is called a **supernova**. All that remains is an explosion nebula and a neutron star ($R \approx 10$ km, $M \approx M_\odot$). The exact mechanism for the transformation of the core implosion to the explosion of the mantle is not fully understood. It is believed to be connected to the mantle's interactions with the SN-neutrinos.
Despite the fact that single Supernovae can be bright as whole galaxies the mantle material and the electromagnetic radiation make up only $\sim 1\%$ of the released energy. Neutrinos carry away the energy bulk.

Supernova Neutrinos We will now focus on the neutrino sector of a SN [15]. As the core collapses, only electron neutrinos are produced via electron capture of nuclei,

$$e^- + N \to \nu_e + N' \,.$$

These neutrinos escape directly, but are not relevant for the overall neutrino luminosity. During the collapse the core density increases and the neutrino's mean-free-path decreases through scattering $N\nu_e \to N\nu_e$. At temperatures higher than 10 MeV and densities over 10^{12}g cm^{-3} the cross-section of neutrino scatterings on heavy nuclei increases [16] such that the neutrinos are trapped. The cross-section of neutrino scatterings is proportional to E_ν^2, only low-energy neutrinos are able to escape the core now. The other neutrinos form the so called 'neutrino sphere' with a radius of

$$R_C \approx 1.0 \times 10^6 \text{cm} \left(\frac{E_\nu}{10 \text{ MeV}} \right) \,.$$

Apart from the early universe this is the only instance where neutrinos are in thermal equilibrium. After the collapse stops and the shock wave emerges all kind of neutrinos can be produced, because thermal processes lead to the presence of relativistic positrons,

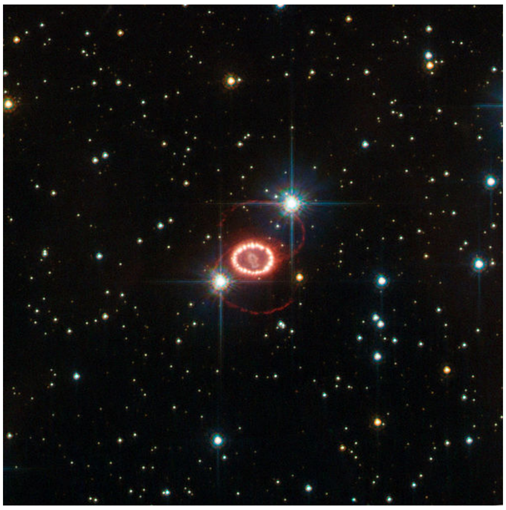
$$\begin{aligned} \gamma\gamma \to e^- e^+ \to \nu_x \bar{\nu}_x \quad &\text{for high } E_\nu \,, \\ NN \to NN\nu_x\bar{\nu}_x \quad &\text{for low } E_\nu \,. \end{aligned}$$

Now the cooling phase occurs, where the hot and dense core emits thermal neutrinos from all flavors. A SN is roughly a black-body source for neutrinos[4].

In our analysis we will simplify this picture by assuming that the core has a radius of 10 km and all flavors of neutrinos are in thermal equilibrium following Fermi-Dirac statistics, see app. E.

[4]Neutrinos of different flavors interact differently with matter. That is why the neutrino sphere is ill-defined [15].

2.2.1. The Observation of SN1987A



On 23 February 1987 the blue giant Sanduleak-69202 in the Large Magellanic Cloud exploded in a supernova that was named SN1987A. Since Kepler's supernova in 1604 it was the first one visible to the naked eye. The remains can be seen in the figure at the side[5]. It is the first and to this day only SN, whose neutrinos have been observed directly. They have been detected in Kamiokande II and IMB [17]. Together with models of gravitational collapse [18] it is possible to estimate the neutrino energy released at SN1987A,

$$E_\nu > 2 \times 10^{53} \text{erg} \,. \tag{2.5}$$

The Argument of Energy Loss Comparing (2.4) with (2.5) we find a correspondence which constrains new physics by an argument of anomalous energy loss [13]. New physics is often accompanied by the presence of new particles. Weakly interacting particles of low mass could contribute to the energy loss or cooling of stars and SNe. These anomalous energy loss mechanisms are however constrained. Comparing again (2.4) and (2.5) a new particle X could only have a SN luminosity of roughly

$$L_X < 10^{52} \frac{\text{erg}}{\text{s}} \,. \tag{2.6}$$

The luminosity L_X gives the integrated energy emitted by anomalous cooling mechanisms via new particles. This argument applies if the new particle escapes from the SN core. It should not diffuse inside the core for longer than $t_{\text{diff}} \sim 1\text{s}$, since the energy gets depleted via neutrino emission during this duration.

[5] http://www.spacetelescope.org/images/potw1142a/

3
Global and Local Supersymmetry

In the first section of this chapter we introduce the general idea of global Supersymmetry in a qualitative way, sketch its development and make the first comments about SUSY breaking.

In section 3.2 we generalize SUSY to a gauge symmetry and describe the idea of Supergravity. Here the gravitino will appear for the first time and we give some details on local SUSY breaking and the Super-Higgs mechanism.

Subsequently we state the full Supergravity Lagrangian in 3.3 as the starting point of our investigations. For this we have to introduce a lot of notations, but we will see explicitly the concepts and ideas from the previous sections. At the end of this chapter we end up with a Lagrangian suitable for our phenomenological studies.

At last we discuss R-Parity and its violations. Here the focus lies on bilinear R-parity violations.

3.1. Global Supersymmetry

The special symmetry between bosons and fermions that we briefly mentioned in ch. 2.1 is called Supersymmetry (SUSY) [10, 19]. The reasons, why SUSY gains a lot of attention by theoretical physicists are diverse, we already mentioned its ability to solve the hierarchy problem. Now we want to add another motivation of a rather aesthetic nature.
It all started in 1971, when it was shown that the Mandula-Coleman theorem [20] could be circumvented. This theorem is based on some very general assumptions and states that the most general Lie algebra of symmetries of the S-matrix of a realistic quantum field theory can only be a direct product of the Poincaré group and a finite number of generators belonging to the Lie algebra of a compact Lie group, which transform as Lorentz scalars. In other words the tensorial symmetries are completely given by the Poincaré generators, i.e. the Lorentz generators $M_{\mu\nu}$ and generators of spatial translations P_μ.
But the assumptions turned out to be too restrictive when it was shown that the Poincaré algebra could be nontrivially extended by generators given by the Weyl-spinors Q_α [21, 22], which obey anti-commutation relations. These spinors[1] satisfy the so-called super-Poincaré algebra,

$$\left\{Q_\alpha, \overline{Q}_{\dot\beta}\right\} = 2\sigma^\mu_{\alpha\dot\beta} P_\mu\,, \tag{3.1}$$

$$\{Q_\alpha, Q_\beta\} = \left\{\overline{Q}_{\dot\alpha}, \overline{Q}_{\dot\beta}\right\} = 0\,, \tag{3.2}$$

$$[Q_\alpha, P^\mu] = \left[\overline{Q}_{\dot\alpha}, P^\mu\right] = 0\,. \tag{3.3}$$

The generators Q satisfying (3.1–3.3) relate bosonic states with fermionic ones,

$$Q|\text{boson}\rangle \sim |\text{fermion}\rangle\,, \quad Q|\text{fermion}\rangle \sim |\text{boson}\rangle\,. \tag{3.4}$$

A finite SUSY transformation is parametrized by a Weyl spinor ϵ and given by $e^{-i\epsilon Q}$. We refer to SUSY as **global**, if ϵ is not spacetime-dependent.
The first supersymmetric, renormalizable field theory interesting for particle physics was formulated by Wess and Zumino in 1974 [23], who presented a supersymmetric

[1] The conventions used for spinors are given in the app. B.

toy model consisting of two real bosonic and one chiral fermionic field. However the first realistic supersymmetric theory, the MSSM, was not introduced until 1981 [24]. Shortly after the work of Wess and Zumino it was shown that SUSY is not just one but the only possible way of extending the symmetry algebra of the S-matrix nontrivially [25]. Of course there are also more phenomenological reasons to study SUSY like the solution of the DM problem mentioned in ch. 2.1 or the unification of gauge couplings at high energies [24].

3.1.1. Global Supersymmetry Breaking

In a supersymmetric theory every bosonic particle in the spectrum has a fermionic partner and vice versa. A complex scalar field, a chiral fermion and an auxiliary field form a so-called supermultiplet (ϕ, ψ, F). In order for a theory to be invariant under SUSY transformations, the bosonic and fermionic superpartners of these supermultiplets must have identical masses. This is obviously excluded. The scalar superpartner of the electron, the selectron, being charged and light, would have been discovered a long time ago. This means that in the true vacuum state $|\Omega\rangle$ of the theory SUSY should be a broken symmetry, leading to a mass gap between the superpartners of one supermultiplet. Since the Hamiltonian of the theory can be expressed in terms of the SUSY generators one can show that

$$Q|\Omega\rangle \neq 0 \Leftrightarrow \langle\Omega|H|\Omega\rangle > 0\,. \tag{3.5}$$

Therefore in order for global SUSY to be spontaneously broken, the vacuum energy must be non-zero. To achieve this one usually assumes that some auxiliary field F obtains a vacuum expectation value (VEV) $\langle F\rangle$. This way supersymmetry is broken at a scale $\Lambda_{\text{SUSY}} = \sqrt{\langle F\rangle}$. After symmetry breaking, a massless Goldstone particle enters the spectrum as in the electroweak theory, but in the case of SUSY this is a fermion, the so-called **goldstino** [21, 26]. As a spin 1/2 particle it has a complex scalar superpartner, the **sgoldstino**, whose mass depends on the specific model.

It turns out that the auxiliary field with the VEV cannot belong to a supermultiplet from the observable field sector, e.g. the electron-selectron supermultiplet. Instead it has to be outsourced to a supermultiplet of a new hidden field sector [27]. This supermultiplet is composed of the goldstino, the sgoldstinos and the auxiliary field F that acquires the VEV. As soon as SUSY breaking occurs in this sector, it gets communicated to the observable parts by certain interactions, which also depend on the SUSY breaking mechanism.

3.2. Local Supersymmetry (Supergravity)

Since the internal symmetries of the SM are all realized locally ('gauged'), it is not far-fetched to examine the consequences of promoting SUSY to a local symmetry. For this the parametrizing spinor ϵ is assumed to be a spacetime dependent function $\epsilon(x)$.

Looking at (3.1) we see that the SUSY generators are connected to the Poincaré group. Hence gauging SUSY also means gauging spacetime translations. Therefore the only way to have a locally supersymmetric theory is to add the Einstein-Hilbert action[2]. The spin-2 graviton has to be part of the particle spectrum and we find that local SUSY is nothing but the combination of supersymmetry and General Relativity (GR) and thus called Supergravity (SUGRA)[3][30, 31].

As every other boson the graviton is part of a supermultiplet and has a fermionic partner of spin $\frac{3}{2}$ called the **gravitino**, whose dynamics is described by the Rarita-Schwinger action. Before SUSY breaking both the graviton and the gravitino are massless. Indeed you can interpret these two fields as the gauge fields of the local Poincaré group and local SUSY respectively. In conclusion this leaves us with the locally supersymmetric part of the action describing the gravity sector,

$$S = \int \mathrm{d}^4 x e \left[-\frac{1}{2\kappa^2} R - \frac{1}{2} \epsilon^{\kappa\lambda\mu\nu} \overline{\psi}_\kappa \gamma^5 \gamma_\lambda \partial_\mu \psi_\nu \right] , \tag{3.6}$$

where e is the determinant of the vielbein (see app. B and C), R is the Ricci scalar, $\kappa = \sqrt{8\pi G_N}$ is the gravitational coupling constant and ψ_μ is the Rarita-Schwinger field describing the gravitino. Since it is the central particle of this thesis we will focus on the gravitino in the separate chapter 4.

[2]We could also argue in the opposite direction and say that SUSY would have to be a local symmetry, since we are sure of gravity's existence.

[3]For a review we refer to [28]. For a pedagogical introduction we also recommend [19, 29].

It is understood that $N = 1$, $d = 4$ SUGRA is not a candidate for a fundamental theory for its non-renormalizability. Just as in the case of perturbative quantization of General Relativity, SUGRA is considered as an effective field theory predictive up to a physical cutoff energy scale.

3.2.1. Local Supersymmetry Breaking and Super-Higgs Mechanism

In sec. 3.1.1 we stated that non-vanishing vacuum energy is the indicator for spontaneously broken supersymmetry in the global case. In SUGRA this is no longer valid and we obtain additional terms in the scalar potential which could cancel the vacuum energy. The Minkowski spacetime would be the theory's classical background. Instead of the vacuum energy the new criterion for SUSY breaking in the local case is given by a non-vanishing gravitino mass $m_{3/2}$. While the graviton remains massless a massive gravitino is a clear indicator of SUSY breaking and directly related to the SUSY breaking scale Λ_{SUSY} via

$$m_{3/2} = \frac{\kappa}{\sqrt{3}} \Lambda^2_{\text{SUSY}} \,. \tag{3.7}$$

But how does the gravitino obtain its mass?
It does in a way similar to the massive vector bosons in electroweak symmetry breaking. In analogy to these particles, the gravitino becomes massive via the Super-Higgs effect [30, 32, 33]. The goldstino becomes its $\pm\frac{1}{2}$ helicity states and disappears from the physical spectrum. This reasoning, which we only sketched here, will be carried out explicitly in the next chapter.
Different SUSY breaking schemes give rise to different values of $m_{3/2}$. For example in Planck-scale mediated SUSY breaking, gravity mediates the symmetry breakdown to the observable sector [34]. In this framework the gravitino typically has a mass comparable to the gaugino masses. There are other models allowing $m_{3/2}$ to be small as we will see in ch. 4.

3.3. General Lagrangian of a Locally Supersymmetric Gauge Theory

The full Lagrangian for a gauge invariant SUGRA Model is derived in [32] and also given in [29]. We start from the Lagrangian given in the app. G of the book by Wess and Bagger and rewrite it using four-component spinors[4]. The full supergravity Lagrangian is given by

$$
\begin{aligned}
e^{-1}\mathcal{L} = & \\
& - \frac{1}{2\kappa^2} R + g_{ij^*} \mathcal{D}_\mu \phi^i \mathcal{D}^\mu \phi^{*j} - \frac{1}{2} g^2 D_{(a)} D^{(a)} + i g_{ij^*} \overline{\chi_L}^j \gamma^\mu \mathcal{D}_\mu \chi_L^i \\
& + e^{-1} \epsilon^{\kappa\lambda\mu\nu} \overline{\psi}_\kappa \gamma_\lambda \mathcal{D}_\mu \psi_{L\nu} - \frac{1}{4} f^R_{(ab)} F^{(a)}_{\mu\nu} F^{(b)\mu\nu} + \frac{1}{8} f^I_{(ab)} \epsilon^{\mu\nu\kappa\lambda} F^{(a)}_{\mu\nu} F^{(b)}_{\kappa\lambda} \\
& + \frac{i}{2} \overline{\lambda}_{(a)} \gamma^\mu \mathcal{D}_\mu \lambda^{(a)} - \frac{1}{2} e^{-1} f^I_{(ab)} \mathcal{D}_\mu \left[e \overline{\lambda}^{(a)} \gamma^\mu \lambda^{(b)}_R \right] - \sqrt{2} g \partial_i D_{(a)} \overline{\lambda}^{(a)} \chi_L^i \\
& - \sqrt{2} g \partial_{j^*} D_{(a)} \overline{\chi_L}^j \lambda^{(a)} + \frac{\kappa}{4} \sqrt{2} g \partial_i f_{(ab)} D^{(a)} \overline{\lambda}^{(b)} \chi_L^i + \frac{\kappa}{4} \sqrt{2} g \partial_{i^*} f^*_{(ab)} D^{(a)} \overline{\chi_L}^i \lambda^{(b)} \\
& + i \frac{\sqrt{2}}{16} \partial_i f_{(ab)} \overline{\lambda}^{(a)} \left[\gamma^\mu, \gamma^\nu\right] \chi_L^i F^{(b)}_{\mu\nu} + i \frac{\sqrt{2}}{16} \partial_{i^*} f^*_{(ab)} \overline{\chi_L}^i \left[\gamma^\mu, \gamma^\nu\right] \lambda^{(a)} F^{(b)}_{\mu\nu} \\
& + \frac{\kappa}{2} g D_{(a)} \overline{\psi}_\mu \gamma^\mu \lambda^{(a)}_R - \frac{\kappa}{2} g D_{(a)} \overline{\psi}_\mu \gamma^\mu \lambda^{(a)}_L - i \frac{\sqrt{2}\kappa}{2} g_{ij^*} \mathcal{D}_\mu \phi^{*j} \overline{\psi}_\nu \gamma^\mu \gamma^\nu \chi_L^i \\
& + i \frac{\sqrt{2}\kappa}{2} g_{ij^*} \mathcal{D}_\nu \phi^i \overline{\chi_L}^j \gamma^\mu \gamma^\nu \psi_\mu - \frac{i\kappa}{16} \left[\overline{\psi}_\mu \left[\gamma^m, \gamma^n\right] \gamma^\mu \lambda_{(a)} \right] \left[F^{(a)}_{mn} + \hat{F}^{(a)}_{mn} \right] \\
& - \frac{\kappa^2}{4} g_{ij^*} \left[i \epsilon^{\kappa\lambda\mu\nu} \overline{\psi}_\kappa \gamma_\lambda \psi_{R\mu} + \overline{\psi}_\mu \gamma^\nu \psi^\mu_R \right] \overline{\chi_L}^j \gamma_\nu {\chi_L}^i \\
& - \frac{\kappa^2}{8} \left(g_{ij^*} g_{kl^*} - 2 R_{ij^*kl^*} \right) \overline{\chi^c_L}^i \chi^k_L \overline{\chi_L}^j \chi^{c\,l}_L \\
& - \frac{1}{16} \left[2\kappa^2 g_{ij^*} f^R_{(ab)} + (f^R_{(cd)})^{-1} \partial_i f_{(bc)} \partial_{j^*} f^*_{(ad)} \right] \overline{\chi_L}^j \gamma^\mu \chi^i_L \overline{\lambda}^{(a)} \gamma_\mu \lambda^{(b)}_L \\
& - \frac{1}{8} \nabla_i \partial_j f_{(ab)} \overline{\chi^c_L}^i \chi^j_L \overline{\lambda}^{(a)} \lambda^{(b)}_L - \frac{1}{8} \nabla_{i^*} \partial_{j^*} f^*_{(ab)} \overline{\chi_L}^i \chi^{c\,j}_L \overline{\lambda}^{(a)} \lambda^{(b)}_R \\
& - \frac{1}{16} (f^R_{(cd)})^{-1} \partial_i f_{(ac)} \partial_j f_{(bd)} \overline{\lambda}^{(a)} \chi^i_L \overline{\lambda}^{(b)} \chi^j_L \\
& - \frac{1}{16} (f^R_{(cd)})^{-1} \partial_{i^*} f^*_{(ac)} \partial_{j^*} f^*_{(bd)} \overline{\chi_L}^i \lambda^{(a)} \overline{\chi_L}^j \lambda^{(b)} \\
& - \frac{1}{16} g^{ij^*} \partial_i f_{(ab)} \partial_{j^*} f^*_{(cd)} \overline{\lambda}^{(a)} \lambda^{(b)}_L \overline{\lambda}^{(c)} \lambda^{(d)}_R + \frac{3\kappa^2}{16} \overline{\lambda}_{(a)} \gamma^\mu \lambda^{(a)}_R \overline{\lambda}_{(b)} \gamma_\mu \lambda^{(b)}_R
\end{aligned}
$$

[4]Note that our conventions, see app. A and B, differ from the ones chosen in [29].

$$
\begin{aligned}
&- \frac{\kappa}{16}\sqrt{2}\partial_i f_{(ab)} \left[\overline{\lambda}^{(a)} \left[\gamma^\mu, \gamma^\nu\right] \chi_L^i \overline{\psi}_\nu \gamma_\mu \lambda_R^{(b)} - \overline{\psi}_\mu \gamma^\mu \chi_L^i \overline{\lambda}^{(a)} \lambda_L^{(b)}\right] \\
&+ \frac{\kappa}{16}\sqrt{2}\partial_{i^*} f^*_{(ab)} \left[\overline{\chi_L}^i \left[\gamma^\mu, \gamma^\nu\right] \lambda^{(a)} \overline{\psi}_\mu \gamma_\nu \lambda_L^{(b)} + \overline{\chi}_L^i \gamma^\mu \psi_\mu \overline{\lambda}^{(a)} \lambda_R^{(b)}\right] \\
&- \exp\left(\frac{\kappa^2 K}{2}\right) \left\{ \frac{\kappa^2}{4} W^* \overline{\psi_{R\alpha}} \left[\gamma^\alpha, \gamma^\beta\right] \psi_{L\beta} + \frac{\kappa^2}{4} W \overline{\psi_{L\alpha}} \left[\gamma^\alpha, \gamma^\beta\right] \psi_{R\beta} \right. \\
&+ \frac{\sqrt{2}\kappa}{2} D_i W \overline{\psi}_\mu \gamma^\mu \chi_L^i + \frac{\sqrt{2}\kappa}{2} D_{i^*} W^* \overline{\chi_L}^i \gamma^\mu \psi_\mu + \frac{1}{2}\mathcal{D}_i D_j W \overline{\chi_L^c}^i \chi_L^j \\
&\left. + \frac{1}{2}\mathcal{D}_{i^*} D_{j^*} W^* \overline{\chi_L}^i \chi_L^{c\,j} + \frac{1}{4} g^{ij^*} D_{j^*} W^* \partial_i f_{(ab)} \overline{\lambda}^{(a)} \lambda_L^{(b)} + \frac{1}{4} g^{ij^*} D_i W \partial_{j^*} f^*_{(ab)} \overline{\lambda}^{(a)} \lambda_R^{(b)} \right\} \\
&- \exp\left(\kappa^2 K\right) \left[g^{ij^*} D_i W (D_j W)^* - 3\kappa^2 W^* W\right] . \qquad (3.8)
\end{aligned}
$$

We will now introduce the individual constituents.

Field Content

Matter Fields

The matter sector consists of chiral superfields. In the component formulation of (3.8) they appear as a lefthanded 4-spinor χ_L^i, given by

$$
\chi_L^i = \begin{pmatrix} (\chi_\alpha^i)_{\mathrm{WB}} \\ 0 \end{pmatrix} , \qquad (3.9)
$$

where $(\chi_\alpha^i)_{\mathrm{WB}}$ is the 2-spinor used by Wess and Bagger in [29], as well as a complex scalar field ϕ^i and an auxiliary field F. The index i runs over all chiral superfields of the respective model and the index α is the spinor index. In the following we will mark all quantities coinciding with the ones in [29] with WB.

Gauge Fields

Since we are interested in gauge invariant models there are also gauge supermultiplets. They consist of a gauge boson $A_\mu^{(a)}$ with the associated field strength tensor

$F^{(a)}_{\mu\nu}$ and a superpartner fermion, the gaugino $\lambda^{(a)}$, written as a Majorana spinor[5],

$$\lambda^{(a)} = \begin{pmatrix} -i\left(\lambda^{(a)}_{\alpha}\right)_{\mathrm{WB}} \\ i\left(\overline{\lambda}^{(a)\dot{\alpha}}\right)_{\mathrm{WB}} \end{pmatrix} , \tag{3.10}$$

and an auxiliary field $D^{(a)}$. The gauge index is always written with brackets, $(a) = 1, 2, ..., \dim \mathcal{G}$ for the gauge group $\mathcal{G}$.

Gravity Fields

In the gravity sector, we have the graviton spin-2 field, given by the vielbein or tetrad e^m_μ that can be found indirectly in the vielbein‘s determinant e and the Ricci scalar R, for more details on the vielbein we refer to the app. B and C. The gravitino is a massless Majorana vector-spinor field ψ_μ,

$$\psi_\mu = \begin{pmatrix} -i\left(\psi_{\mu\alpha}\right)_{\mathrm{WB}} \\ i\left(\overline{\psi}^{\dot{\alpha}}_{\mu}\right)_{\mathrm{WB}} \end{pmatrix} . \tag{3.11}$$

In contrast to the gauge and matter fields there are two auxiliary fields, a complex scalar field M and a real vector field b^μ, which secure a consistent off-shell description of the gravity sector. These do not appear in (3.8) because they have already been eliminated using their respective field equation.

Auxiliary Fields

The auxiliary fields of the chiral supermultiplets F^i do not appear in the Lagrangian for the same reason. Their equation of motion reads

$$F^i = e^{\kappa^2 K/2} g^{ij^*} D_{j^*} W^* . \tag{3.12}$$

We will need the field equations for the scalar M field given by

$$M = -3\kappa e^{\kappa^2 K/2} W . \tag{3.13}$$

[5] Just as in [35] we add additional factors of i in the definition of the gaugino and gravitino fields.

The $D^{(a)}$ fields have not been eliminated. However their field equations are given by

$$D^{(a)} = \mathrm{Re}\left(f^{-1}\right)_{(ab)} K_i \left(t^{(a)}\phi\right)^i , \tag{3.14}$$

where the generators $t^{(a)}$ of the Lie algebra of the gauge group $\mathcal{G}$ occur for the first time.

Matter Coupling to the SUGRA multiplet

The coupling of matter to the gravity sector of the model is encoded in three functions:

1. The first is the **superpotential** W, an analytic function of the scalar fields ϕ^i as well as scalar fields h from a hidden sector,

$$W(\phi^i, h) = W_h(h) + W_o(\phi^i) . \tag{3.15}$$

   It has a mass dimension of 3 and determines the self-interactions of the scalar fields and their Yukawa couplings. The scalar field h plays an essential role when it comes to SUSY breaking. We will have to come back to this in sec. 3.3.

2. The **Kähler potential** $K(\phi^i)$ is a real function of mass dimension 2, that appears either directly in (3.8) or indirectly in the Kähler metric

$$g_{ij^*} := \frac{\partial^2 K}{\partial\phi^i \partial\phi^{*j}} . \tag{3.16}$$

   It is in particular responsible for the kinetic term of the scalar fields. The corresponding Kähler connection and curvature are given by

$$\Gamma^k_{ij} = g^{kl^*} \frac{\partial g_{jl^*}}{\partial\phi^i} , \tag{3.17}$$

$$R_{ij^*kl^*} = g_{ml^*} \frac{\partial}{\partial\phi^{*j}} \Gamma^m_{ik} . \tag{3.18}$$

3. The **gauge kinetic function** $f_{(ab)}(\phi^i)$ is located in front of the kinetic terms of the gauge boson and gauginos. It is a dimensionless and analytic function. Its real part, together with its real inverse, is used to raise and lower the gauge indices.

In this context we introduce the notation

$$\partial_i f_{(ab)} := \frac{\partial f_{(ab)}}{\partial \phi^i}\,, \quad \partial_{i^*} f_{(ab)} := \frac{\partial f_{(ab)}}{\partial \phi^{*i}}\,, \tag{3.19}$$

$$K_i := \frac{\partial K}{\partial \phi^i}\,, \quad \text{and} \quad K_{i^*} := \frac{\partial K}{\partial \phi^{*i}}\,. \tag{3.20}$$

Covariant Derivatives and SUSY Transformation

The covariant derivatives in (3.8) and (3.12) are given by

$$\mathcal{D}_\mu \phi^i = \partial_\mu \phi^i + igA_\mu^{(a)} g^{ij^*} \partial_{j^*} D_{(a)}\,, \tag{3.21}$$

$$\begin{aligned}\mathcal{D}_\mu \chi_L^i = {} & \partial_\mu \chi_L^i + \frac{i}{4}\omega_\mu{}^{ab}\sigma_{ab}\chi_L^i + \Gamma^i_{jk}\mathcal{D}_\mu\phi^j\chi_L^k + igA_\mu^{(a)}\partial_k\left(g^{ij^*}\partial_{j^*}D_{(a)}\right)\chi_L^k \\ & - \frac{1}{4}\kappa^2\left(K_j\mathcal{D}_\mu\phi^j - K_{j^*}\mathcal{D}_\mu\phi^{*j}\right)\chi_L^i - \frac{i}{2}\kappa^2 gA_\mu^{(a)}\,\mathrm{Im}\,F_{(a)}\chi_L^i\,, \end{aligned} \tag{3.22}$$

$$\begin{aligned}\mathcal{D}_\mu \lambda^{(a)} = {} & \partial_\mu \lambda^{(a)} + \frac{i}{4}\omega_\mu{}^{ab}\sigma_{ab}\lambda^{(a)} - gf^{(abc)}A_\mu^{(b)}\lambda^{(c)} \\ & + \frac{1}{4}\kappa^2\left(K_j\mathcal{D}_\mu\phi^j - K_{j^*}\mathcal{D}_\mu\phi^{*j}\right)\lambda^{(a)} + \frac{i}{2}\kappa^2 gA_\mu^{(b)}\,\mathrm{Im}\,F_{(b)}\lambda^{(a)}\,, \end{aligned} \tag{3.23}$$

$$\begin{aligned}\mathcal{D}_\mu \psi_\nu = {} & \partial_\mu \psi_\nu + \frac{i}{4}\omega_\mu{}^{ab}\sigma_{ab}\psi_\nu \\ & + \frac{1}{4}\kappa^2\left(K_j\mathcal{D}_\mu\phi^j - K_{j^*}\mathcal{D}_\mu\phi^{*j}\right)\psi_\nu + \frac{i}{2}\kappa^2 gA_\mu^{(a)}\,\mathrm{Im}\,F_{(a)}\psi_\nu\,, \end{aligned} \tag{3.24}$$

$$D_i W = W_i + \kappa^2 K_i W\,, \tag{3.25}$$

$$\begin{aligned}\mathcal{D}_i D_j W = {} & W_{ij} - \Gamma^k_{ij} D_k W \\ & + \kappa^2\left(K_{ij}W + K_i D_j W + K_j D_i W - \kappa^2 K_i K_j W\right)\,. \end{aligned} \tag{3.26}$$

The Lagrangian is invariant under the local SUSY transformation parametrized by the Majorana spinor $\xi(x) = \begin{pmatrix}\xi_{WB}(x)\\ \bar{\xi}_{WB}(x)\end{pmatrix}$,

$$\delta_\xi e_\mu{}^a = \kappa\left(\bar{\xi}\gamma^a\psi_{R\mu} - \bar{\xi}\gamma^a\psi_{L\mu}\right)\,, \tag{3.27}$$

$$\delta_\xi \phi^i = \sqrt{2} i \bar{\xi} \chi^i_L \,, \tag{3.28}$$

$$\begin{aligned}\delta_\xi \chi^i_L = &-i\sqrt{2}\gamma^\mu \xi_R \mathcal{D}_\mu \phi^i - \Gamma^i_{jk}\delta_\xi \phi^j \chi^k_L - \sqrt{2} e^{\kappa^2 K/2} g^{ij^*} D_{j^*} W^* \xi_L \\ &- \frac{1}{4}\sqrt{2}\xi_L g^{ij^*}\partial_{j^*} f^*_{(ab)} \overline{\lambda}^{(a)} \lambda^{(b)}_R + \frac{1}{4}\kappa^2 \left(K_j \delta_\xi \phi^j - K_{j^*}\delta_\xi \phi^{*j}\right)\chi^i_L \,,\end{aligned} \tag{3.29}$$

$$\delta_\xi A^{(a)}_\mu = \bar{\xi}\gamma_\mu \lambda^{(a)}_R - \bar{\xi}\gamma_\mu \lambda^{(a)}_L \,, \tag{3.30}$$

$$\begin{aligned}\delta_\xi \lambda^{(a)}_L = &\frac{1}{4} F^{(a)}_{\mu\nu}\left[\gamma^\mu, \gamma^\nu\right]\xi_L - igD^{(a)}\xi + \frac{i}{4}\sqrt{2}\xi (f^R_{(ab)})^{-1}\partial_i f_{(bc)} \overline{\lambda}^{(c)} \chi^i_L \\ &+ \frac{i}{4}\sqrt{2}\xi (f^R_{(ab)})^{-1}\partial_{i^*} f^*_{(bc)} \overline{\chi}^i_L \lambda^{(c)} - \frac{i}{4}\kappa^2 (K_j \delta_\xi \phi^j - K_{j^*}\delta_\xi \phi^{*j})\lambda^{(a)}_L \,,\end{aligned} \tag{3.31}$$

$$\begin{aligned}\delta_\xi \psi_\mu = &\frac{2}{\kappa}\mathcal{D}_\mu \xi + i\kappa e^{\kappa^2 K/2} W \gamma_\mu \xi_R - \frac{i}{4}\kappa^2 (K_j \delta_\xi \phi^j - K_{j^*}\delta_\xi \phi^{*j})\psi_{L\mu} \\ &- \frac{i}{8}\kappa\left[\gamma_\mu, \gamma_\nu\right]\xi g_{ij^*}\overline{\chi}^j_L \gamma^\nu \chi^i_L - \frac{i}{2}\kappa\left(g_{\mu\nu} + \frac{1}{4}\left[\gamma^\mu, \gamma^\nu\right]\right)\xi\, \overline{\lambda}_{(a)}\gamma^\nu \lambda^{(a)}_R \,.\end{aligned} \tag{3.32}$$

Here we also used the covariant derivative of ξ given by

$$\mathcal{D}_\mu \xi = \partial_\mu \xi + \frac{i}{4}\omega_\mu{}^{ab}\sigma_{ab}\xi + \frac{1}{4}\kappa^2\left(K_j \mathcal{D}_\mu \phi^j - K_{j^*}\mathcal{D}_\mu \phi^{*j}\right)\xi \,. \tag{3.33}$$

SUSY Breaking and Super-Higgs Effect

In the ch. 3.1.1 and 3.2.1 we discussed many aspects of SUSY breaking in a qualitative way. Now we want to carry out the corresponding steps explicitly.
If SUSY is spontaneously broken, the superpartner of the Standard-Model particles could indeed attain a mass high enough to explain that they remain unobserved to this day.
We already saw that **global** SUSY is broken, if and only if the vacuum energy $\langle\Omega| H |\Omega\rangle = \langle\Omega| V |\Omega\rangle$ is non-zero. Here V is the scalar potential. In global SUSY it is given by

$$V(\phi) = F^i F^{*i} + \frac{1}{2}D^{(a)}D^{(a)} \,. \tag{3.34}$$

Note that this potential is always positive or zero. If one of the auxiliary fields F^i attains a vacuum expection value (VEV) $\langle F^i \rangle$, we will obtain a non-vanishing vacuum energy and thus spontaneous breaking of supersymmetry[6]. With a positive

[6] At this point we will not consider the possiblity of SUSY breaking via a non-vanishing $\langle D^{(a)}\rangle$. For reasons see e.g. [10]

vacuum energy, we would obtain de-Sitter spacetime as our classical background. Now let us turn to **local** supersymmetry. In this case the scalar potential (3.34) is generalized to

$$V(\phi) = F^i g_{ij^*} F^{*j^*} + \frac{g^2}{2} \operatorname{Re} f_{(ab)} D^{(a)} D^{(b)} - \frac{1}{3} M^* M \,. \tag{3.35}$$

After implementing the gravity sector the potential is no longer positive semi-definite. Since a significant cosmological constant is experimentally disfavored we demand that the vacuum energy vanishes,

$$\langle V \rangle = 0 \quad \Rightarrow \quad \sqrt{\langle F^i g_{ij^*} F^{*j^*} \rangle} = \frac{1}{\sqrt{3}} \langle |M| \rangle \,. \tag{3.36}$$

In contrast to a globally supersymmetric model, we can choose flat Minkowski spacetime as our classical background.

Gravitino Mass

We define the supersymmetry breaking scale as $\Lambda^4_{SUSY} \equiv \langle F^i g_{ij^*} F^{*j^*} \rangle$ and evaluate (3.7),

$$m_{3/2} = \frac{\kappa \Lambda^2_{SUSY}}{\sqrt{3}} = \frac{\kappa}{\sqrt{3}} \sqrt{\langle F^i g_{ij^*} F^{*j^*} \rangle} = \frac{\kappa}{3} \langle |M| \rangle \,. \tag{3.37}$$

With (3.13) we obtain

$$m_{3/2} = \kappa^2 e^{\kappa^2 \langle K \rangle / 2} \langle |W| \rangle \,. \tag{3.38}$$

This combination appears in the Lagrangian (3.8) giving rise to the gravitino mass term,

$$\begin{aligned} & - e \exp\left(\frac{\kappa^2 K}{2}\right) \left(\frac{\kappa^2}{4} W^* \overline{\psi_{R\alpha}} \left[\gamma^\alpha, \gamma^\beta\right] \psi_{L\beta} + \text{h.c.}\right) \\ \Longrightarrow & - \frac{1}{4}\, e\, m_{3/2}\, \overline{\psi}_\alpha \left[\gamma^\alpha, \gamma^\beta\right] \psi_\beta \,. \end{aligned} \tag{3.39}$$

The gravitino is now massive. As outlined in sec. 3.2.1 it obtained its two additional degrees of freedom by absorbing the goldstino, which can be set to zero. In other words, the goldstino-gravitino mixing term in (3.8),

$$-\frac{i\sqrt{2}\kappa}{2}\langle D_i W\rangle \overline{\chi_L^c}^i \gamma^\alpha \psi_{R\alpha} + \text{h.c.}\,, \tag{3.40}$$

vanishes.

Gaugino Masses

The only term in (3.8) that could generate gaugino masses is

$$-ee^{\kappa^2 K/2}\left[\frac{1}{4}g^{ij^*}D_{j^*}W^*\partial_i f_{(ab)}\overline{\lambda_R^{(a)}}\lambda_L^{(b)} + \text{h.c.}\right]\,. \tag{3.41}$$

Based on these terms we define the gaugino mass matrix,

$$M_{(ab)} = \frac{1}{2}e^{\kappa^2\langle K\rangle/2}g^{ij^*}\langle D_{j^*}W\rangle\langle\partial_i f_{(ab)}\rangle\,, \tag{3.42}$$

which gives the possibility of mixing the gaugino's mass eigenstates.

3.3.1. Minimal Choices for the Lagrangian

In the following we will only consider terms up to the order κ^1 and neglect terms of higher order with one exception that we discuss later.
In addition we have to make some choices for the Kähler potential, the gauge kinetic function and the superpotential. We choose them minimally, just in order to get the desired features of the theory, e.g. gaugino masses and canonical kinetic terms,

$$f_{(ab)} = \delta_{(ab)}f(h)\,, \tag{3.43}$$

$$K(h,h^*,\phi^i,\phi^{*i}) = K_h(h,h^*) + \phi^i\phi^{*i}\,, \tag{3.44}$$

$$W(\phi,h) = W_h(h) + W_o(\phi^i)\,. \tag{3.45}$$

We assume that the function $f(h)$ is real. Note that $\partial_i f_{(ab)}$ is now of the order κ^1. Other consequences of this choice are a trivial Kähler metric, (3.16) becomes

$g_{ij^*} = \delta_{ij^*}$, vanishing connection terms $\Gamma^k_{ij} = 0$ as well as a vanishing Kähler curvature $R_{ij^*kl^*} = 0$. Furthermore we do no longer distinguish between upper and lower gauge indices $(a), (b), ...$ or superfield indices $i, j, ...$, since they are now raised and lowered by $\delta_{(ab)}$ and δ_{ij^*} respectively. Furthermore the gaugino mass matrix (3.42) is diagonal.

$$\begin{aligned}
e^{-1}\mathcal{L} = &-\frac{1}{2\kappa^2}R + \mathcal{D}_\mu\phi^i\mathcal{D}^\mu\phi^{*i} - \frac{1}{2}g^2(f(h))^{-1}D^{(a)}D^{(a)} + i\overline{\chi_L}^i\gamma^\mu\mathcal{D}_\mu\chi^i_L \\
&+ e^{-1}\epsilon^{\kappa\lambda\mu\nu}\overline{\psi}_\kappa\gamma_\lambda\mathcal{D}_\mu\psi_{L\nu} - \frac{1}{4}f(h)F^{(a)}_{\mu\nu}F^{(a)\mu\nu} + \frac{i}{2}f(h)\overline{\lambda}^{(a)}\gamma^\mu\mathcal{D}_\mu\lambda^{(a)} \\
&+ \Bigg[-\sqrt{2}g\partial_i D_{(a)}\overline{\lambda}^{(a)}\chi^i_L + \frac{1}{4}\sqrt{2}gf(h)^{-1}\partial_i f(h)D^{(a)}\overline{\lambda}^{(a)}\chi^i_L \\
&+ i\frac{\sqrt{2}}{16}\partial_i f(h)\overline{\lambda}^{(a)}\left[\gamma^\mu,\gamma^\nu\right]\chi^i_L F^{(a)}_{\mu\nu} + \frac{\kappa}{2}gf(h)^{-1}D^{(a)}\overline{\psi}_\mu\gamma^\mu\lambda^{(a)}_R \\
&- i\frac{\sqrt{2}\kappa}{2}\mathcal{D}_\mu\phi^{*i}\overline{\psi}_\nu\gamma^\mu\gamma^\nu\chi^i_L + \text{h.c.}\Bigg] - \frac{i\kappa}{8}f(h)\left[\overline{\psi}_\mu\left[\gamma^m,\gamma^n\right]\gamma^\mu\lambda^{(a)}\right]F^{(a)}_{mn} \\
&- \exp\left(\frac{\kappa^2 K}{2}\right)\Bigg[\frac{\kappa^2}{4}W^*\overline{\psi_{R\alpha}}\left[\gamma^\alpha,\gamma^\beta\right]\psi_{L\beta} + \frac{\kappa}{\sqrt{2}}D_iW\overline{\psi}_\mu\gamma^\mu\chi^i_L \\
&\qquad + \frac{1}{2}\mathcal{D}_iD_jW\overline{\chi^c_L}^i\chi^j_L + \frac{1}{4}D_{i^*}W^*\partial_i f_{(ab)}\overline{\lambda}^{(a)}\lambda^{(b)}_L + \text{h.c.}\Bigg] \\
&- \exp\left(\kappa^2 K\right)\left[D_iW(D_iW)^* - 3\kappa^2W^*W\right] \\
&- \frac{\kappa^2}{4}\left[i\epsilon^{\kappa\lambda\mu\nu}\overline{\psi}_\kappa\gamma_\lambda\psi_{R\mu} + \overline{\psi}_\mu\gamma^\nu\psi^\mu_R\right]\overline{\chi_L}^i\gamma_\nu{\chi_L}^i + \mathcal{O}(\kappa^2)\,. \qquad (3.46)
\end{aligned}$$

The last term may be of order κ^2, nonetheless it will be relevant in one situation later on. The auxiliary D fields are simply given by

$$D_{(a)} = \phi^{*i}\,t^{(a)}_{ij}\,\phi^j\,. \qquad (3.47)$$

After SUSY breaking the field h from the hidden sector acquires a VEV $\langle h\rangle$ (as well as $\langle f\rangle, \langle W_h\rangle, \langle K_h\rangle$) and we obtain the following gravitino and gaugino masses,

$$\begin{aligned}
m_{3/2} &= \kappa^2 e^{\kappa^2\langle K_h\rangle/2}\langle|W_h|\rangle\,, \\
M_{(ab)} &= \frac{1}{2}e^{\kappa^2\langle K_h\rangle/2}\langle D_{i^*}W_h\rangle\langle\partial_i f(h)\rangle\delta_{(ab)}\,.
\end{aligned}$$

We do not want the function $f(h)$ to appear explicitly in our Lagrangian, so we perform the following re-scaling as it was done in [35],

$$\lambda \longrightarrow \hat{\lambda} = \sqrt{\langle f \rangle}\lambda\,, \tag{3.48}$$

$$A_\mu^{(a)} \longrightarrow \hat{A}_\mu^{(a)} = \sqrt{\langle f \rangle} A_\mu^{(a)}\,, \tag{3.49}$$

$$g \longrightarrow \hat{g} = \frac{g}{\sqrt{\langle f \rangle}}\,, \tag{3.50}$$

$$M_{(ab)} \longrightarrow \hat{M}_{(ab)} = \frac{M_{(ab)}}{\langle f \rangle}\,. \tag{3.51}$$

We immediately abbreviate the notation and omit the hats. In consequence of the re-scaling we obtain canonically normalized gauge kinetic terms. These choices leave us with the Lagrangian,

$$\begin{aligned}
e^{-1}\mathcal{L} = &-\frac{1}{2\kappa^2}R + \mathcal{D}_\mu\phi^i\mathcal{D}^\mu\phi^{*i} - \frac{1}{2}g^2(\phi^{*i}t_{ij}^{(a)}\phi^j)(\phi^{*k}t_{kl}^{(a)}\phi^l) + i\overline{\chi_L}^i\gamma^\mu\mathcal{D}_\mu\chi_L^i \\
&- \frac{1}{2}e^{-1}\epsilon^{\kappa\lambda\mu\nu}\overline{\psi}_\kappa\gamma^5\gamma_\lambda D_\mu\psi_\nu + \frac{i}{2}m_{3/2}\overline{\psi}_\alpha\sigma^{\alpha\beta}\psi_\beta - \frac{1}{4}F_{\mu\nu}^{(a)}F^{(a)\mu\nu} \\
&+ \frac{i}{2}\overline{\lambda}^{(a)}\gamma^\mu\mathcal{D}_\mu\lambda^{(a)} - \frac{1}{2}M_{(a)}\left(\overline{\lambda}_R^{(a)}\lambda_L^{(a)} + \text{h.c.}\right) \\
&+ \Bigg[-\sqrt{2}g(\phi^{*j}t_{ji}^{(a)})\overline{\lambda}^{(a)}\chi_L^i + \frac{\kappa}{2}g(\phi^{*i}t_{ij}^{(a)}\phi^j)\overline{\psi}_\mu\gamma^\mu\lambda_R^{(a)} \\
&- i\frac{\sqrt{2}\kappa}{2}\mathcal{D}_\mu\phi^{*i}\overline{\psi}_\nu\gamma^\mu\gamma^\nu\chi_L^i + \text{h.c.}\Bigg] - \frac{i\kappa}{8}\left[\overline{\psi}_\mu\left[\gamma^m,\gamma^n\right]\gamma^\mu\lambda^{(a)}\right]F_{mn}^{(a)} \\
&- \exp\left(\frac{\kappa^2 K}{2}\right)\Bigg[\frac{\kappa^2}{4}W^*\overline{\psi_{R\alpha}}\left[\gamma^\alpha,\gamma^\beta\right]\psi_{L\beta} + \frac{\kappa}{\sqrt{2}}D_iW\overline{\psi}_\mu\gamma^\mu\chi_L^i \\
&\qquad + \frac{1}{2}\mathcal{D}_iD_jW\overline{\chi_L^c}^i\chi_L^j + \frac{1}{4}D_{i^*}W^*\partial_i f_{(ab)}\overline{\lambda}^{(a)}\lambda_L^{(b)} + \text{h.c.}\Bigg] \\
&- \exp\left(\kappa^2 K\right)\left[D_iW(D_iW)^* - 3\kappa^2W^*W\right] \\
&- \frac{\kappa^2}{4}\left[i\epsilon^{\kappa\lambda\mu\nu}\overline{\psi}_\kappa\gamma_\lambda\psi_{R\mu} + \overline{\psi}_\mu\gamma^\nu\psi_R^\mu\right]\overline{\chi_L}^i\gamma_\nu{\chi_L}^i + \mathcal{O}(\kappa^2)\,.
\end{aligned} \tag{3.52}$$

The masses of chiral fermions can be produced as usual from the bilinear part of the superpotential, the Yukawa couplings emerge from trilinear terms [35]. In ch. 4.3 this Lagrangian will be our starting point.

3.4. R-Parity

In this section we will assume the field content of the Minimal Supersymmetric Standard Model (MSSM)[7]. The superpotential of the MSSM is given by

$$W_{\mathrm{MSSM}} = \mu H_u H_d + \lambda^e_{ij} H_d L_i E^c_j + \lambda^d_{ij} H_d Q_i D^c_j - \lambda^u_{ij} H_u Q_i U^c_j \,, \tag{3.53}$$

where the indices i, j, k run over the three generations. However these terms are not the most general set of renormalizable and gauge invariant terms.
Demanding gauge invariance and renormalizability the SM contains the most general set of Yukawa couplings. This naturally leads to the conservation of baryon number B and lepton number L, whose underlying symmetry can be regarded as 'accidental'. This nice feature is lost once we add SUSY. It is possible to generalize B and L to the sparticles, but some of the couplings compatible with gauge invariance and renormalizability are very problematic. Namely the superpotential terms proportional to

$$L \cdot L\, E^c \,, \qquad Q \cdot L\, D^c \,, \qquad U^c D^c D^c \,, \tag{3.54}$$

that cannot be found in (3.53), lead to undesirable features. They introduce new effective four-fermion interactions via exchange of squarks or other bosons that do not conserve B or L. These interactions could spoil our model containing electromagnetic, weak and strong interactions via gauge spin-1 boson mediation. They can also lead to rapid proton decay, whose life time is very strictly bounded from below ($\tau_P > 2.1 \cdot 10^{29} y$ [36]).
One solution was found by Pierre Fayet, who introduced a new discrete symmetry, called R-parity [37][8] and given by

$$R_P = (-1)^{3B+L+2s} = \begin{cases} +1 & \text{for particles,} \\ -1 & \text{for sparticles}\,. \end{cases} \tag{3.55}$$

[7] For an introduction we refer for example to [10, 19].
[8] For a pedagogical introduction we refer to [19]. For a review on R-Parity and its violation we recommend [38].

The conservation of R-parity forbids the terms in (3.54) and allows the necessary Yukawa couplings of the Higgs-fields present in (3.53), because $R_P = -1$ for Q, LU^c, D^c, E^c and $R_P = +1$ for H_u and H_d.
It also means that the lightest supersymmetry particle (LSP) is stable and that sparticles can only be produced in pairs. R-parity could emerge as the discrete remnant of a broken global U(1) symmetry, whose generators R do not commute with the SUSY generators,

$$[Q_\alpha, R] = Q_\alpha \,, \quad \left[\overline{Q}_{\dot{\alpha}}, R\right] = -\overline{Q}_{\dot{\alpha}} \,, \tag{3.56}$$

and therefore treats particles and sparticles differently. This kind of internal global symmetry is present in N=1 SUSY [19] but in the end one still has to impose R-parity conservation as an additional input of the theory.

3.4.1. Bilinear R-Parity Violations

It should be clear by now that there is no fundamental reason to not include the superpotential terms

$$W_{\not{R}} = \mu_i H_u \cdot L_i + \frac{1}{2}\lambda_{ijk} L_i \cdot L_j E_k^c + \lambda'_{ijk} L_i \cdot Q_j D_k^c + \frac{1}{2}\lambda''_{ijk} U_i^c D_j^c D_k^c \,. \tag{3.57}$$

However, as said before, these couplings are highly constrained phenomenologically since they might lead to violation of baryon and lepton number. But only if both B and L are violated simultaneously the proton could decay via e.g. $p \longrightarrow \pi^0 e^+$ and it is possible to introduce lepton number violating terms compatible with empirical data [39].
We will relax the MSSM's assumption of invariance under R-parity and allow **bilinear R-parity violations** [38, 40] given by the first term in (3.57),

$$W_{\not{R}_P} = \mu_i H_u \cdot L_i \,. \tag{3.58}$$

This term is motivated by its ability to generate a hierarchical neutrino mass spectrum favored by observations (e.g. the solar neutrino problem) [41].
After SUSY breaking additional terms containing bilinear RPV can be found in

the soft SUSY-breaking terms,

$$\mathcal{L}_{\text{soft}} = \left(B_i H_u \tilde{L}_i + \tilde{m}^2_{di} H^\dagger_d \tilde{L}_i + \text{h.c.} \right) + ... \tag{3.59}$$

We can include the nine R-parity violating parameters $\mu_i, B_i, \tilde{m}^2_{di}$ while respecting existing experimental bounds since baryon number is still conserved and the proton remains stable. This is not at all an unnatural thing to do and can change the predicted phenomena completely [42].
Without R-parity we can no longer distinguish between the Higgsino H_d and leptons L_i. Mixing becomes a possibility and we can perform rotations of the weak eigenstates [39]

$$\begin{pmatrix} H_d \\ L_i \end{pmatrix} \mapsto \begin{pmatrix} H'_d \\ L'_i \end{pmatrix} = U \begin{pmatrix} H_d \\ L_i \end{pmatrix} , \quad \text{with } U \in SU(4) . \tag{3.60}$$

This rotations lead to a tranformation of the RPV parameters such as

$$\mu_i \mapsto \mu'_i = U^*_{i0} \mu + U^*_{ij} \mu_j , \tag{3.61}$$

and also generates new Yukawa couplings λ_{ijk} and λ'_{ijk}[9]. Due to the bilinear term (3.59) the sneutrinos $\tilde{\nu}_i$ typically acquire a VEV $v_i = \frac{\langle \tilde{\nu}_i \rangle}{\sqrt{2}}$ after radiative electroweak symmetry breaking alongside with the Higgs VEVs v_u and v_d [43]. But we should note that the value of the sneutrino VEV depends on the choice of weak interaction basis and changes under the transformation (3.60) as

$$v_i \mapsto v'_i = U_{i0} v_d + U_{ij} v_j . \tag{3.62}$$

Of course it does not matter which basis is chosen. We could find weak eigenstates such that $v_i = 0$ and $\mu_i \neq 0$. Following [39] we choose,

$$U = \begin{pmatrix} 1 & -\epsilon_i \\ \epsilon^*_i & \mathbb{1}_{3\times 3} \end{pmatrix} , \quad \text{for } \epsilon_i \equiv \frac{\mu_i}{\mu} \ll 1 . \tag{3.63}$$

[9]These new trilinear terms do not lead to baryon number violations.

In this basis the three parameter μ_i vanish and trilinear $\not{R}$ terms emerge in the superpotential. We have a non-vanishing sneutrino VEV given by [40]

$$\frac{v_i}{v_d} \approx \frac{B_i \tan\beta - m^{*2}_{H_d L_i}}{\tilde{m}^2_{\tilde{l}_{ij}} + \frac{1}{2} m_Z \cos 2\beta}\,. \tag{3.64}$$

The angle β is defined by $\tan\beta \equiv \frac{v_u}{v_d}$.

Neutrino-Neutralino and Chargino-Charged Lepton Mixing

As mentioned before without lepton number conservation bilinear RPV gives rise to new mixings. Especially the neutralinos mix with the neutrinos and the charginos mix with the charged leptons. These interactions are of special relevance to us, since they lead to new effective gravitino couplings to leptons, as we will see in chapter 4.4.

The 4×4 neutralino mixing matrix in the MSSM becomes a 7×7 matrix M^7_N given in a basis $(-i\tilde{\gamma},\ -i\tilde{Z}^0,\ \tilde{H}^0_u,\ \tilde{H}^0_d,\ \nu_i)^T$. This matrix can be diagonalized using a unitary 7×7 matrix N^7,

$$\begin{pmatrix} \tilde{\chi}^0_1 \\ \tilde{\chi}^0_2 \\ \tilde{\chi}^0_3 \\ \tilde{\chi}^0_4 \\ \tilde{\chi}^0_{4+i} \end{pmatrix} = N^7 \begin{pmatrix} -i\tilde{\gamma} \\ -i\tilde{Z}^0 \\ \tilde{H}^0_u \\ \tilde{H}^0_d \\ \nu_i \end{pmatrix}. \tag{3.65}$$

Following [40] we define the photino-zino, zino-zino and higgsino-zino mixing parameters,

$$U_{\tilde{\gamma}\tilde{Z}} = m_Z \sum_{i=1}^{4} \frac{N^*_{i\tilde{\gamma}} N_{i\tilde{Z}}}{m_{\tilde{\chi}^0_i}} \approx m_Z \sin\theta_W \cos\theta_W \frac{M_2 - M_1}{M_1 M_2}\,, \tag{3.66}$$

$$U_{\tilde{Z}\tilde{Z}} = m_Z \sum_{i=1}^{4} \frac{N^*_{i\tilde{Z}} N_{i\tilde{Z}}}{m_{\tilde{\chi}^0_i}} \approx -m_Z \left(\frac{\sin^2\theta_W}{M_1} + \frac{\cos^2\theta_W}{M_2} \right), \tag{3.67}$$

$$U_{\tilde{H}^0_u\tilde{Z}} = m_Z \sum_{i=1}^{4} \frac{N^*_{i\tilde{H}^0_u} N_{i\tilde{Z}}}{m_{\tilde{\chi}^0_i}} \approx m_Z^2 \cos\theta_W \frac{M_1 \cos^2\theta_W + M_2 \sin^2\theta_W}{M_1 M_2 \mu}\,, \tag{3.68}$$

$$U_{\tilde{H}_d^0 \tilde{Z}} = m_Z \sum_{i=1}^{4} \frac{N^*_{i\tilde{H}_d^0} N_{i\tilde{Z}}}{m_{\tilde{\chi}_i^0}} \approx -m_Z^2 \sin\theta_W \frac{M_1 \cos^2\theta_W + M_2 \sin^2\theta_W}{M_1 M_2 \mu} \,. \tag{3.69}$$

Here we wrote down the leading terms in the expansion of N^7 in $\xi \equiv \frac{v_i}{v}$ (with $v = v_u + v_d$) and m_Z.

4

Phenomenology of Superlight Gravitinos

We now focus on the central particle of this thesis. After a short discussion of the Rarita-Schwinger action, which describes the free massive spin-$\frac{3}{2}$ gravitino field, we explain why a superlight gravitino is of special interest for particle physics phenomenology. This also involves the SUSY equivalence theorem stating that a very light gravitino effectively acts as a massless spin-$\frac{1}{2}$ goldstino.

In sec. 4.3 we complete the derivation of our model and state the Feynman rules necessary for our investigations. Also we comment briefly on the complex role of the sgoldstino fields and their couplings and masses.

The focus of the final section lies on the additional gravitino interactions arising from bilinear RPV terms in the superpotential. The vacuum expectation value of a sneutrino field gives new effective couplings between gravitino, matter fermions and gauge bosons.

4.1. Free Gravitinos – The Rarita-Schwinger-Field

The Lagrangian of a free massive spin $\frac{3}{2}$ field already appears in (3.52). It can be written [44] as

$$\mathcal{L} = -\frac{1}{2}\epsilon^{\mu\nu\kappa\lambda}\overline{\psi}_\mu \gamma^5 \gamma_\nu \partial_\kappa \psi_\lambda - \frac{1}{4} m_{3/2} \overline{\psi}_\mu \left[\gamma^\mu, \gamma^\nu\right] \psi_\nu \,. \tag{4.1}$$

The corresponding field equations are the Rarita-Schwinger equations [45],

$$\epsilon^{\mu\nu\kappa\lambda}\gamma^5 \gamma_\nu \partial_\kappa \psi_\lambda + \frac{1}{2} m_{3/2} \left[\gamma^\mu, \gamma^\nu\right] \psi_\nu = 0 \,. \tag{4.2}$$

Alternatively they can be expressed in the form

$$\partial^\mu \psi_\mu(x) = 0 \,, \tag{4.3}$$

$$\gamma^\mu \psi_\mu(x) = 0 \,, \tag{4.4}$$

$$(i\partial\!\!\!/ - m_{3/2})\psi_\mu(x) = 0 \,. \tag{4.5}$$

Therefore the object ψ_μ is a four-vector with spinors as entries, each satisfying the Dirac equation. Hence the solution $\psi_\mu \sim e^{-ipx}\tilde{\psi}_\mu$ can be written using a spin-$\frac{1}{2}$ spinor u and a polarization vector ϵ_μ of a spin 1 field,

$$\tilde{\psi}_\mu(\vec{p}, \lambda) = \sum_{s,m} \left\langle \left(\frac{1}{2}, s\right) (1, m) \,|\, \left(\frac{3}{2}, \lambda\right) \right\rangle u(\vec{p}, s) \epsilon_\mu(\vec{p}, m) \,, \tag{4.6}$$

where $\left\langle \left(\frac{1}{2}, s\right) (1, m) \,|\, \left(\frac{3}{2}, \lambda\right) \right\rangle$ are Clebsch-Gordan coefficients [46].
When summing over the gravitino spins we will need the polarization tensor of ψ_μ with momentum p,

$$\begin{aligned}\Pi^{\pm}_{\mu\nu}(k) &= \sum_s \psi^{\pm,s}_\mu(k) \overline{\psi}^{\pm,s}_\nu(k) \quad , \left(\text{where } s = \pm\frac{1}{2}, \pm\frac{3}{2}\right) \\ &= -(p\!\!\!/ \pm m_{3/2}) \times \\ &\quad \left[\left(g_{\mu\nu} - \frac{p_\mu p_\nu}{m^2_{3/2}}\right) - \frac{1}{3}\left(g_{\mu\sigma} - \frac{p_\mu p_\sigma}{m^2_{3/2}}\right)\left(g_{\nu\lambda} - \frac{p_\lambda p_\nu}{m^2_{3/2}}\right)\gamma^\sigma \gamma^\lambda\right] ,\end{aligned} \tag{4.7}$$

for the positive and negative frequency solution respectively [40, 44]. For a derivation we refer for example to [47].
The polarization tensor satisfies the following relations,

$$\gamma^\mu \Pi^\pm_{\mu\nu}(p) = \Pi^\pm_{\mu\nu}(p)\gamma^\nu = 0\,, \tag{4.8}$$

$$p^\mu \Pi^\pm_{\mu\nu}(p) = \Pi^\pm_{\mu\nu}(p)p^\nu = 0\,, \tag{4.9}$$

$$(\not{p} - m_{3/2})\Pi^\pm_{\mu\nu}(p) = \Pi^\pm_{\mu\nu}(p)(\not{p} - m_{3/2}) = 0\,. \tag{4.10}$$

4.2. Phenomenology of Superlight Gravitinos

The actual value of the gravitino mass heavily depends on the SUSY breaking scheme. For example in Planck-scale mediated SUSY breaking gravity communicates the symmetry breakdown to the observable sector [34]. In this framework the gravitino typically has a mass comparable to the gaugino masses. For phenomenological reasons we are interested in superlight gravitinos only, which do not appear in the framework of Planck-scale mediated SUSY breaking. However there are other models allowing $m_{3/2}$ to be small like certain no-scale models [48] and models with gauge-mediated SUSY breaking (GMSB) [49]. In the latter gauge interactions communicate the SUSY breaking to the observable matter fields of our theory leading to the necessary mass gaps. Gravity, being present as well of course, is not relevant in this context. But why should we restrict ourselves to superlight gravitinos?
In 1977 the French physicist Pierre Fayet came to the conclusion that a superlight gravitino would be very favourable from a phenomenologist's point of view [50]. He concludes

> "[...]that the super-Higgs mechanism gives to the gravitino, and to gravitation effects in particle physics, their chance to be detected, since weak interactions can be generated from gravitational ones."

A small value of $m_{3/2}$ would enhance the gravitino's interactions that would have been otherwise suppressed by a prefactor of the gravitational coupling κ. It turns out e.g. that the effective gravitino-gauge boson-gaugino vertex is proportional to $\kappa \frac{m_{\tilde{V}}}{m_{3/2}}$, the magnitude of the fraction $\frac{m_{\tilde{V}}}{m_{3/2}}$ could compensate the weakness of

gravity.
We already saw that the massive gravitino obtains its $\pm\frac{1}{2}$ helicity states by absorbing the goldstino. If the gravitino mass is very small compared to the energy scale of the relevant process, these helicity states dominate and the gravitino effectively behaves like the massless goldstino, while its $\pm\frac{3}{2}$ helicity states are negligible. This is called the SUSY Equivalence Theorem [51]. In the equivalence theorem limit we can write

$$\psi_\mu \sim i\sqrt{\frac{2}{3}}\frac{1}{m_{3/2}}\partial_\mu\psi\,, \tag{4.11}$$

where ψ is the spin-$\frac{1}{2}$ goldstino. In certain models this enhancement of gravitational interactions affects also the sgoldstinos. In contrast to the goldstino, the sgoldstinos do not disappear from the physical spectrum and can be very light [52, 53]. Therefore a superlight gravitino could open not only a phenomenological door to gravitation but also to the observation of new particles from a hidden sector, which we will discuss further in sec. 4.3.

Indeed our analysis is based on the assumption that the gravitino mass is very small. But in using these simplifications we implicitly neglect terms in the gravitino polarization tensor (4.7). In cross-sections of reactions with more than one external gravitino these terms might be relevant and cannot be ignored. Instead the whole polarization tensor should be used. In order to simplify our calculations we can expand (4.7) in powers of $m_{3/2}$,

$$\begin{aligned}\Pi^{\pm}_{\mu\nu}(k) &= \frac{2}{3}\frac{k_\mu k_\nu}{m_{3/2}^2}\not{k} \pm \frac{1}{3}\frac{4k_\mu k_\nu - k_\mu\not{k}\gamma_\nu - k_\nu\gamma_\mu\not{k}}{m_{3/2}} \\ &\quad + \frac{1}{3}\left(-3g_{\mu\nu}\not{k} + \not{k}\gamma_\mu\gamma_\nu - \gamma_\nu k_\mu + \gamma_\mu k_\nu\right) + \mathcal{O}(m_{3/2}) \\ &\equiv \frac{1}{m_{3/2}^2}\Pi_{(2)\mu\nu}(k) \pm \frac{1}{m_{3/2}}\Pi_{(1)\mu\nu}(k) + \Pi_{(0)\mu\nu}(k) + \mathcal{O}(m_{3/2})\,.\end{aligned} \tag{4.12}$$

Note that the number of γ-matrices is even for $\Pi_{(1)}$ and odd for $\Pi_{(2)}$ and $\Pi_{(0)}$.
In practice, when dealing with superlight gravitinos, it often will turn out to be sufficient to retain the leading order in (4.12), i.e. $\Pi_{(2)\mu\nu}(k)$ and to substitute (4.11). Before doing so one should perform a careful power counting of $m_{3/2}$ in the

occurring amplitudes. We will elaborate on this in sec. 5.2.1.

Previous Studies

Various approaches to test the possibility of a superlight and detectable gravitino were proposed. Possible experimental settings are colliders: e^+e^- annihilation [53, 54], $\gamma\gamma$ annihilation [55, 56] and hadron collider [57] were considered to constrain a superlight gravitino. In this thesis we want to use observations from astrophysics and compare them to the predictions derived from particle physics models. But also within astroparticle physics there have been various approaches to a superlight gravitino. Some early constraints on its mass were derived from cosmology, more precisely from Big Bang Nucleosynthesis (BBN). It was shown that BBN allows either a light gravitino < 1keV or a very heavy one [58]. These investigations have been revisited in 1993 by Moroi [59] and four years later again by Gherghetta [60, 61]. The latter found that $m_{3/2} \gtrsim$ 1eV. Other approaches consisted of the possibility of exotic cooling of stars, red giants and white dwarfs [62]. These were reviewed in [63].

4.3. Gravitino Interactions and Feynman-Rules

Now we will derive the final Lagrangian of our model, that we will use in the ch. 5 in the case of conserved R-parity. It is a broken-supersymmetric Einstein-Maxwell system with the gauge group $U(1)_{\mathrm{QED}}$. We extract all necessary terms from (3.52) and read off the Feynman rules, especially the ones of the Gravitino interactions. For this gauge group we only have one gaugino, the massive photino ($M_{(ab)} \to m_{\tilde{\gamma}}$). The relevant kinetic terms and Gravitino interactions are contained in the following Lagrangian,

$$\begin{aligned} e^{-1}\mathcal{L} = &- \frac{1}{2\kappa^2}R - \frac{1}{2}e^{-1}\epsilon^{\kappa\lambda\mu\nu}\overline{\psi}_\kappa\gamma^5\gamma_\lambda D_\mu\psi_\nu + \frac{i}{2}m_{3/2}\overline{\psi}_\alpha\sigma^{\alpha\beta}\psi_\beta - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\ &+ \frac{i}{2}\overline{\lambda}^{(a)}\left[\gamma^\mu D_\mu - m_{\tilde{\gamma}}\right]\lambda^{(a)} + D_\mu\phi^i D^\mu\phi^{*i} - m_{\phi_i}\phi^{*i}\phi^i \\ &+ i\overline{\chi^i}_L\gamma^\mu D_\mu\chi^i_L - \frac{1}{2}m_{\chi_i}\left(\overline{\chi^c_L}^i\chi^i_L + \mathrm{h.c.}\right) \\ &- \frac{i\kappa}{\sqrt{2}}\left(D_\mu\phi^{*i}\overline{\psi}_\nu\gamma^\mu\gamma^\nu\chi^i_L - D_\mu\phi^i\overline{\chi}^i_L\gamma^\nu\gamma^\mu\psi_\nu\right) - \frac{\kappa}{4}\overline{\psi}_\mu\sigma^{\rho\sigma}\gamma^\mu\lambda\, F_{\rho\sigma} \\ &- \frac{\kappa^2}{4}\left[ie^{-1}\epsilon^{\kappa\lambda\mu\nu}\overline{\psi}_\kappa\gamma_\lambda\psi_{R\mu} + \overline{\psi}_\mu\gamma^\nu\psi^\mu_R\right]\overline{\chi_L}^i\gamma_\nu{\chi_L}^i + \mathcal{O}(\kappa^2)\,. \end{aligned} \tag{4.13}$$

The covariant derivatives are given by

$$\begin{aligned} D_\mu\phi^i &= \partial_\mu\phi^i + iQ_iA_\mu\phi^i\,, \quad D_\mu\chi^i_L = \partial_\mu\chi^i_L + \frac{i}{4}\omega_{\mu ab}\sigma^{ab}\chi^i_L + iQ_iA_\mu\chi^i_L\,, \\ D_\mu\lambda &= \partial_\mu\lambda + \frac{i}{4}\omega_{\mu ab}\sigma^{ab}\lambda\,, \quad D_\mu\psi_\nu = \partial_\mu\psi_\nu + \frac{i}{4}\omega_{\mu ab}\sigma^{ab}\psi_\nu\,. \end{aligned}$$

Here Q_i is the charge of the respective field[1].

Sgoldstino Couplings and Masses

The couplings of the scalar field h from the hidden sector to matter depend largely on the specific choices for the h-depending parts of the gauge kinetic function, the Kähler potential and the superpotential (3.43)-(3.44). The model we choose has been derived in [32, 64] and employed by various authors in the context of gravitino phenomenology [1, 55, 56, 60, 61, 65]. It inherits a canonical Kähler potential,

[1]Note that we do not sum over the index i.

a vanishing cosmological constant and D-term SUSY breaking. The Lagrangian involving the sgoldstino reads

$$\begin{aligned} e^{-1}\mathcal{L}_{\text{Sgoldstino}} &= \frac{1}{2}\left(\partial_\mu S\partial^\mu S - m_S S^2 + \partial_\mu P\partial^\mu P - m_P P^2\right) \\ &+ \frac{\kappa}{4} c F_{\mu\nu}F^{\mu\nu}S + i\frac{\kappa}{2}m_{3/2}d\overline{\psi}_\mu\sigma^{\mu\nu}\psi_\nu S \\ &- \frac{\kappa}{8}ce^{-1}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}P - i\frac{\kappa}{4}d\epsilon^{\mu\nu\rho\sigma}\overline{\psi}_\mu\gamma_\nu\psi_\rho\partial_\sigma P + \mathcal{O}(\kappa^2)\,. \end{aligned} \tag{4.14}$$

The scalar and pseudo-scalar fields S/P are the real sgoldstino components [55] given by

$$S = \frac{1}{\sqrt{2}}\left(h + h^*\right)\,, \quad P = \frac{1}{\sqrt{2}i}\left(h - h^*\right)\,. \tag{4.15}$$

The dimensionless couplings c and d emerge from the super-Higgs mechanism and are related to each other,

$$c \cdot d = \frac{m_\lambda}{m_{3/2}}\,, \tag{4.16}$$

or more specifically for no-scale models [55],

$$c = -\sqrt{\frac{2}{3}}\frac{m_{\tilde{\gamma}}}{m_{3/2}}\,, \quad d = -\sqrt{\frac{3}{2}}\,. \tag{4.17}$$

We do not need to include the sgoldstino's couplings to other fermions like the neutrinos because they are not getting amplified by the factor $m_{3/2}^{-1}$ [65].
In the end all these assumptions still do not fix the sgoldstino masses. They depend on possible additional terms in the Kähler potential [61] and could be very light, such as the gravitino, or very heavy. The phenomenology of these cases differs dramatically and we will distinguish the two cases in our analysis in ch. 5.

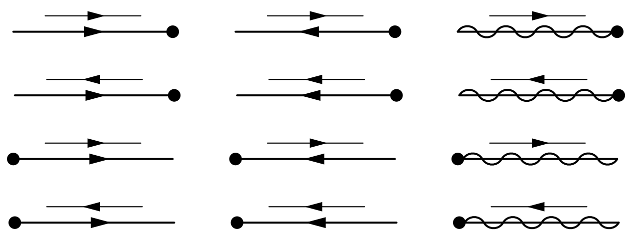
Feynman Rules

We will now present the Feynman rules of our model. Our model contains Majorana fermions like the photino and the gravitino. We will handle Majorana spinors by using the method of a continuous fermion flow [66]. Depending on the fermion flow direction, we obtain two expression for each vertex.

External Lines

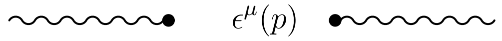
The momentum p flows from left to right.

- Matter Fermions and Photino:



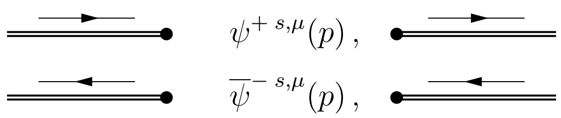
$$u^s(p)\,,\quad \overline{v}^s(p)\,,\quad \overline{u}^s(p)\,,\quad v^s(p)\,.$$

- Gauge bosons:



$$\mu \quad \epsilon^{\mu}(p) \qquad \mu \quad \epsilon^{*\mu}(p)\,.$$

- Gravitinos:



$$\mu \quad \psi^{+\,s,\mu}(p)\,, \qquad \mu \quad \overline{\psi}^{+\,s,\mu}(p)\,,$$
$$\mu \quad \overline{\psi}^{-\,s,\mu}(p)\,, \qquad \mu \quad \psi^{-\,s,\mu}(p)\,.$$

When we use the equivalence theorem (4.11) the more precise relations read

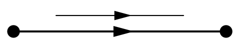
$$\psi^{+\,s,\mu}(p) \approx i\sqrt{\frac{2}{3}}\frac{p^{\mu}}{m_{3/2}}u^s(p)\,, \tag{4.18}$$

$$\psi^{-\,s,\mu}(p) \approx i\sqrt{\frac{2}{3}}\frac{p^{\mu}}{m_{3/2}}v^s(p)\,. \tag{4.19}$$

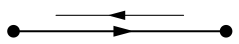
Usually we will suppress the spin index s for convenience.

Propagators

- Matter fermions:



$$\frac{i(\not{p}+m_\chi)}{p^2-m_\chi^2}\,,$$



$$\frac{i(-\not{p}+m_\chi)}{p^2-m_\chi^2}\,.$$

- Matter scalars:

$$\frac{i}{p^2-m_\phi^2}\,.$$

- Gauge boson (in the $\xi = 1$ gauge):

μ ν

$$\frac{-ig_{\mu\nu}}{p^2-m_A^2}\,.$$

- Gaugino:

$$\frac{i(\not{p}+m_\lambda)}{p^2-m_\lambda^2}\,.$$

- Graviton (see app. C)

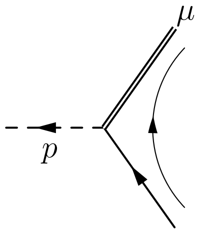
$\mu\nu$  $\alpha\beta$

$$\frac{i}{2p^2}\left(\eta_{\mu\alpha}\eta_{\nu\beta}+\eta_{\mu\beta}\eta_{\nu\alpha}-\eta_{\mu\nu}\eta_{\alpha\beta}\right)\,.$$

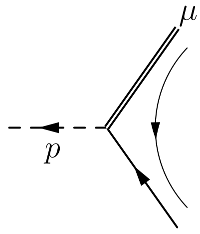
Vertices from (4.13)

We present the Feynman rules relevant for the upcoming analysis. For a complete set (without the Graviton rules) we refer to [35]. For the graviton vertices we have to linearize gravity. The derivation can be found in the app. C.

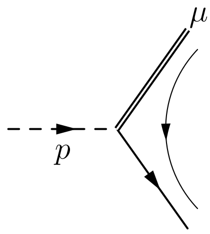
All momenta flow into the vertex.



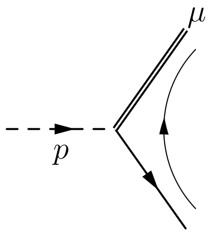
$\frac{i\kappa}{\sqrt{2}}\not{p}\gamma_\mu P_L$,



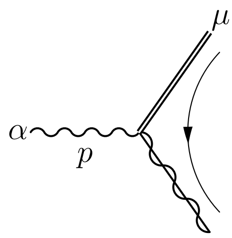
$\frac{i\kappa}{\sqrt{2}}P_L\gamma_\mu\not{p}$,



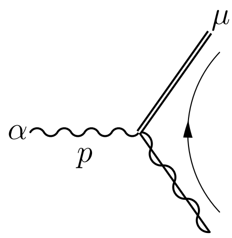
$-\frac{i\kappa}{\sqrt{2}}P_R\gamma_\mu\not{p}$,



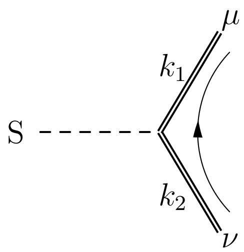
$-\frac{i\kappa}{\sqrt{2}}\not{p}\gamma_\mu P_R$,



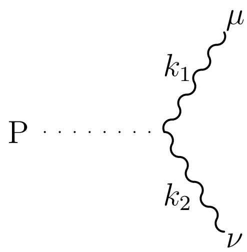
$-\frac{\kappa}{2}\gamma^\mu\sigma_{\alpha\beta}p^\beta$,



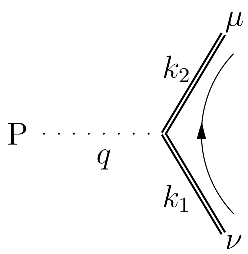
$-\frac{\kappa}{2}\sigma_{\alpha\beta}p^\beta\gamma_\mu$,



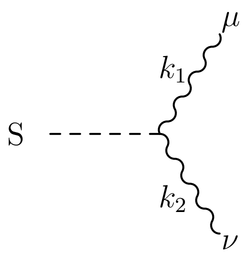
$i\kappa d m_{3/2}\eta_{\mu\nu}$,



$i\kappa c\epsilon_{\sigma\rho\mu\nu}k_1^\sigma k_2^\rho$,

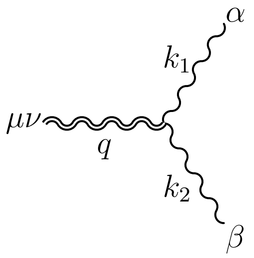


$\frac{i\kappa d}{2}\epsilon_{\mu\sigma\nu\rho}\gamma^\sigma q^\rho$,

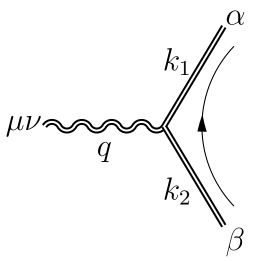


$- i\kappa c \times$

$[(k_1 \cdot k_2)\eta_{\mu\nu} - k_{1\nu}k_{2\mu}]$,



$$2i\kappa\Big[\eta_{\alpha(\mu}k_{2\nu)}k_{1\beta}+\eta_{\beta(\mu}k_{1\nu)}k_{2\alpha}-(k_1\cdot k_2)\eta_{\alpha(\mu}\eta_{\nu)\beta}-\eta_{\alpha\beta}k_{1(\mu}k_{2\nu)} + \frac{1}{2}\eta_{\mu\nu}\left((k_1\cdot k_2)\eta_{\alpha\beta}-k_{1\beta}k_{2\alpha}\right)\Big],$$

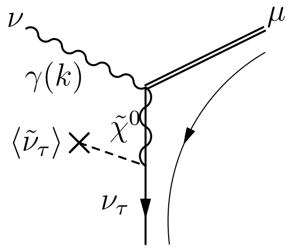


$$\frac{\kappa}{2}\Big[\epsilon_{\alpha\sigma\beta(\mu}\gamma^5\gamma_{\nu)}(k_2-k_1)^\sigma+\frac{i}{2}\epsilon_{\alpha\sigma\beta(\mu}\gamma^5\left\{\gamma^\sigma,\sigma_{\nu)\rho}\right\}q^\rho + 2im_{3/2}\left(\eta_{\alpha\beta}\eta_{\mu\nu}-2\eta_{\alpha(\mu}\eta_{\nu)\beta}\right)\Big].$$

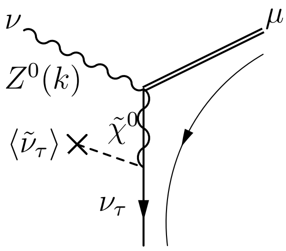
These vertices agree with the one given in [35] and [55]. Although the authors of the former employ the Veltman definition of the graviton field (C.9) leading to some extra factors of $\sqrt{2}$, for more details we again refer to the app. C of this thesis.

4.4. R-Parity Violating Gravitino Vertices

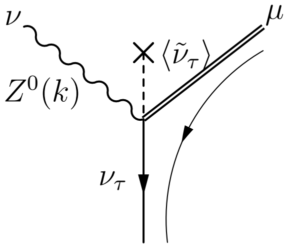
In sec. 3.4 we introduced R-parity and bilinear R-parity violations. We mentioned that models including bilinear RPV can inherit a sneutrino VEV $\langle\tilde{\nu}\rangle$ and also allow for neutrinos and neutralinos to mix. This leads to effective interactions between neutrinos, gravitinos and gauge bosons.



$$= \frac{i\kappa g_Z\langle\tilde{\nu}\rangle}{8\sqrt{2}m_Z}U_{\tilde{\gamma}\tilde{Z}}\left(1+\gamma^5\right)\gamma_\mu\left[\not{k},\gamma_\nu\right], \tag{4.20}$$



$$= \frac{i\kappa g_Z \langle \tilde{\nu} \rangle}{8\sqrt{2} m_Z} U_{\tilde{Z}\tilde{Z}} \left(1 + \gamma^5\right) \gamma_\mu \left[\not{k}, \gamma_\nu \right] , \tag{4.21}$$



$$= -\frac{i g_Z \kappa \langle \tilde{\nu} \rangle}{4\sqrt{2}} \left(1 + \gamma^5\right) \gamma^\mu \gamma^\nu . \tag{4.22}$$

These diagrams have been computed in the context of gravitino decays [40, 47, 67]. We have to estimate these parameters such that we are able to make quantitative statements later on.

Instead of the sneutrino VEV we will use the parameter $\xi \equiv \frac{\langle \tilde{\nu} \rangle}{v}$, where v is the SM-Higgs VEV of $v \equiv (\sqrt{2} G_F)^{-1/2} \approx 246$ GeV. For the estimation of the gravitino luminosity later on we will assume

$$\xi \sim 10^{-7} \tag{4.23}$$

such that neutrino masses below 1 eV can be obtained. The mixing parameter defined in (3.66) and (3.67) can be estimated roughly if neutralino masses are of the same order $M_1 \sim M_2 \sim M_{1/2}$ [40].

$$U_{\tilde{\gamma}\tilde{Z}} = U_{\tilde{Z}\tilde{Z}} \approx \frac{m_Z}{M_{1/2}} , \quad \text{and } M_{1/2} = \mathcal{O}(100\text{GeV}) , \tag{4.24}$$

$$\approx 0.8 . \tag{4.25}$$

At last the electroweak coupling parameter is defined as

$$g_Z \equiv \frac{g}{\cos\theta_W} , \quad \text{with } g = 32^{1/4} \sqrt{G_F} M_W ,$$
$$\approx 0.65 . \tag{4.26}$$

5

Supernova Constraints on Superlight Gravitinos

The first part of our analysis is about gravitino pair production in supernovae with conserved R-Parity. We review in great detail some results from the mid-90s [1] and obtain similar astrophysical bounds on the gravitino mass.

In particular these authors indirectly assume to have massless sgoldstinos in their model which enhance the gravitino pair production heavily. We perform the same calculation with very massive sgoldstinos and obtain strongly altered results. This shows the strong model-dependence of these bounds.

In sec. 5.3 we change the initial particles from photons to neutrinos and show that this gives rise to additional relevant supernova cooling channels once the assumption of massless sgoldstinos is dismissed. Yet, this process does not lead to qualitatively new findings.

5.1. Gravitino Pair Luminosity

The luminosity for gravitinos produced via $\gamma(p_1)\gamma(p_2) \longrightarrow \tilde{G}(k_1)\tilde{G}(k_2)$ is given [68] by

$$\begin{aligned} L = V \int \frac{\mathrm{d}^3 p_1}{(2\pi)^3\, 2p_1^0}\, 2n_\gamma(p_1^0) \int \frac{\mathrm{d}^3 p_2}{(2\pi)^3\, 2p_2^0}\, 2n_\gamma(p_2^0) \int \frac{\mathrm{d}^3 k_1}{(2\pi)^3\, 2k_1^0} \int \frac{\mathrm{d}^3 k_2}{(2\pi)^3\, 2k_2^0} \\ (2\pi)^4 \delta^{(4)}(p_1 + p_2 - k_1 - k_2)(k_1^0 + k_2^0)\overline{\left|\mathcal{M}(\gamma\gamma \to \tilde{\mathcal{G}}\tilde{\mathcal{G}})\right|^2}\,. \end{aligned} \tag{5.1}$$

Thus, the luminosity is given by the overall amount of gravitino energy produced inside a volume V with temperature T via the collision of photons in thermal equilibrium. The temperature enters the luminosity via the photon Bose-Einstein distribution function $n_\gamma(p_i^0)$, given by

$$n_\gamma(p_i^0) = \frac{1}{e^{p_i^0/T} - 1} \quad \text{(see app. E)}\,.$$

For the squared amplitude $\overline{|\mathcal{M}|^2}$, we already averaged over incoming and summed over outgoing spins. Now we can use energy conservation, the expression for the relative velocity (D.18) and the total cross-section (D.20),

$$\begin{aligned} L &= \frac{V}{(2\pi)^6} \int \mathrm{d}^3 p_1 2n_\gamma(p_1^0) \int \mathrm{d}^3 p_2 2n_\gamma(p_2^0)(p_1^0 + p_2^0)|v_1 - v_2| N_{\mathrm{id}}!\sigma(\gamma\gamma \to \tilde{G}\tilde{G}) \\ &= \frac{8V}{(2\pi)^6} \int \mathrm{d}^3 p_1 n_\gamma(p_1^0) \int \mathrm{d}^3 p_2 n_\gamma(p_2^0)(p_1^0 + p_2^0) \frac{p_1 \cdot p_2}{p_1^0 p_2^0} \sigma(\gamma\gamma \to \tilde{G}\tilde{G})\,, \end{aligned} \tag{5.2}$$

where N_{id} is the number of identical particles in the final states appearing in (D.20)[1]. In order to simplify the integration we use that $n_\gamma(p_i^0) > e^{-p_i^0/T}$ for all p^0. Hence,

$$L > \frac{8V}{(2\pi)^6} \int \mathrm{d}^3 p_1 \mathrm{d}^3 p_2 e^{-(p_1^0 + p_2^0)/T}(p_1^0 + p_2^0)\,(1 - \cos\alpha)\,\sigma(\gamma\gamma \longrightarrow \tilde{G}\tilde{G})\,, \tag{5.3}$$

where $\alpha = \measuredangle(\vec{p}_1, \vec{p}_2)\,.$

The cross-section in general depends on the photons' momenta or rather on the Mandelstam variable s. We express the photon momenta p_1 and p_2 in spherical

[1] This expression differs from the luminosity given in [1] by a factor of 8.

coordinates $(p_i^0, \theta_i, \phi_i)$ and obtain

$$s = (p_1 + p_2)^2 = 2p_1^0 p_2^0 (1 - \cos\alpha)\,, \tag{5.4}$$

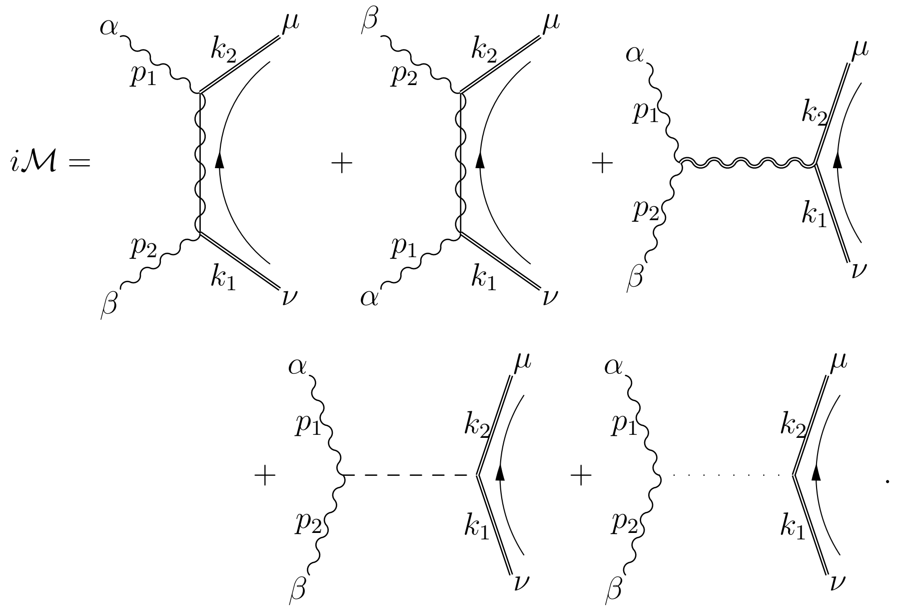
$$\text{and } \cos\alpha = \sin\theta_1 \sin\theta_2 \cos(\phi_1 - \phi_2) + \cos\theta_1 \cos\theta_2\,. \tag{5.5}$$

The next step is to calculate $\sigma(\gamma\gamma \longrightarrow \tilde{G}\tilde{G})$.

5.2. Gravitino Pair Production via Photon Collision

5.2.1. Calculation of the Cross-Section

We consider the process $\gamma(p_1)\gamma(p_2) \to \tilde{G}(k_1)\tilde{G}(k_2)$. The contributing diagrams are given by



(5.6)

Applying the Feynman rules given in sec. 4.2 and app. C, we obtain the amplitude of this process,

$$i\mathcal{M} = i\mathcal{M}_{\text{Photino}} + i\mathcal{M}_{\text{Graviton}} + i\mathcal{M}_{\text{Scalar}} + i\mathcal{M}_{\text{Pseudoscalar}}\,, \tag{5.7}$$

where

$$
\begin{aligned}
i\mathcal{M}_{\text{Photino}} &= \frac{i\kappa^2}{4}\,\epsilon_1^\alpha\epsilon_2^\beta\,p_1^\kappa p_2^\lambda\quad \overline{\psi}^{+\,\mu}(k_2)\sigma_{\alpha\kappa}\gamma_\mu\frac{\not{q}_1 - m_{\tilde{\gamma}}}{q_1^2 - m_{\tilde{\gamma}}^2}\gamma_\nu\sigma_{\beta\lambda}\psi^{-\,\nu}(k_1) \\
&\quad + \frac{i\kappa^2}{4}\,\epsilon_2^\alpha\epsilon_1^\beta\,p_2^\kappa p_1^\lambda\quad \overline{\psi}^{+\,\mu}(k_2)\sigma_{\alpha\kappa}\gamma_\mu\frac{\not{q}_2 - m_{\tilde{\gamma}}}{q_2^2 - m_{\tilde{\gamma}}^2}\gamma_\nu\sigma_{\beta\lambda}\psi^{-\,\nu}(k_1)\,, \qquad (5.8) \\
i\mathcal{M}_{\text{Graviton}} &= \frac{\kappa^2}{2(p_1+p_2)^2}\Big((\epsilon_1\cdot\epsilon_2)p_1^\lambda p_2^\rho + \frac{1}{2}\big((p_1\cdot\epsilon_2)(p_2\cdot\epsilon_1) - (p_1\cdot p_2)(\epsilon_1\cdot\epsilon_2)\big)\eta^{\lambda\rho} \\
&\quad + (p_1\cdot p_2)\epsilon_1^\lambda\epsilon_2^\rho - (p_2\cdot\epsilon_1)\epsilon_2^\rho p_1^\lambda - (p_1\cdot\epsilon_2)p_2^\lambda\epsilon_1^\rho + (\rho\leftrightarrow\lambda)\Big) \\
&\quad \overline{\psi}^{+\,\mu}(k_2)\Big[\epsilon_{\mu\sigma\nu(\lambda}\gamma^5\gamma_{\rho)}(k_2-k_1)^\sigma + \frac{i}{2}\epsilon_{\mu\sigma\nu(\lambda}\gamma^5\left\{\gamma^\sigma,\sigma_{\rho)\tau}\right\} \\
&\qquad - 2im_{3/2}(2\eta_{\mu(\lambda}\eta_{\rho)\nu} - \eta_{\mu\nu}\eta_{\lambda\rho})\Big]\psi^{-\,\nu}(k_1)\,, \qquad (5.9) \\
i\mathcal{M}_{\text{Scalar}} &= \frac{i\kappa^2 m_{\tilde{\gamma}}}{(p_1+p_2)^2 - m_S^2}\times \\
&\quad \epsilon_1^\alpha\epsilon_2^\beta\,\big((p_1\cdot p_2)\eta_{\alpha\beta} - p_1^\beta p_2^\alpha\big)\,\eta_{\mu\nu}\,\overline{\psi}^{+\,\mu}(k_2)\psi^{-\,\nu}(k_1)\,, \qquad (5.10) \\
i\mathcal{M}_{\text{PseudoScalar}} &= -\frac{i\kappa^2 m_{\tilde{\gamma}}}{2m_{3/2}}\,\frac{1}{(p_1+p_2)^2 - m_P^2}\times \\
&\quad \epsilon_1^\alpha\epsilon_2^\beta\,p_1^\kappa p_2^\lambda\,\epsilon_{\kappa\lambda\alpha\beta}\,(p_1+p_2)^\zeta\,\epsilon_{\mu\delta\nu\zeta}\,\overline{\psi}^{+\,\mu}(k_2)\gamma^\delta\psi^{-\,\nu}(k_1)\,. \qquad (5.11)
\end{aligned}
$$

After squaring (5.7) we have to sum over the spins of the photons and gravitinos. For the gravitinos we could just use (4.7). It is however beneficial to first detect the relevant terms in the polarization tensor and neglect the rest.

Identifying the Relevant Terms – Power-Counting of $\mathbf{m_{3/2}}$

Using the equivalence theorem (see sec. 4.2) can be a neat way of simplifying the calculation of the cross-section. Nevertheless it cannot be used without caution, since we deal with a process involving two external gravitinos. In this case it turns out that we would miss important terms of leading order of $m_{3/2}$ if we simply use (4.18-4.19). Instead we perform a careful power-counting before we can exploit the equivalence theorem.

In the case of gravitino pair production a generic amplitude is given by

$$i\mathcal{M}_1 = \frac{1}{m_{3/2}^{i_1}} \overline{\psi}^{+\,\mu}(k_2)\Theta_{1,\mu\nu}^{(n_1,\ldots,n_N)}\psi^{-\,\nu}(k_1)\,. \tag{5.12}$$

The amplitude consists of N terms with n_j γ-matrices respectively, indicated by the indices $(n_1, ..., n_N)$. We will obtain several contributing diagrams and therefore several amplitudes. After averaging over initial and summing over final states, a general term in the squared amplitude reads

$$\overline{\left|\mathcal{M}_1^\dagger \mathcal{M}_2\right|} = \frac{1}{m_{3/2}^{i_1+i_2}} \operatorname{Tr}\left[\Pi^{\nu\nu'}(k_1)\tilde{\Theta}_{1,\mu'\nu'}^{(n_1,\ldots,n_N)}\Pi^{\mu'\mu}(k_2)\Theta_{2,\mu\nu}^{(m_1,\ldots,m_M)}\right]. \tag{5.13}$$

The first step in our calculation will be the identification of the leading order $i_{leading}$ of $m_{3/2}$. For this we proceed as follows:

1. We find the amplitudes with the leading order i_{max} of $m_{3/2}$.
2. We use the equivalence theorem and calculate the square of these amplitudes. The result should be $\propto m_{3/2}^{-(2i_{max}+4)}$.
3. Now there are two possibilities:
   a) The result does not vanish, $i_{leading} = 2i_{max} + 4$. In this case we are done, since the other diagrams will only give higher order corrections to the cross-section.
   b) The result vanishes. Now the second term in (4.12) could possibly yield the leading terms of order $m_{3/2}^{-(2i_{max}+3)}$ and cannot be neglected. Also other diagrams with lower power $i_{max} - 1$ of $m_{3/2}$ become relevant again and may not be neglected as there are interference terms of the order $m_{3/2}^{-(4+i_{max}+(i_{max}-1))}$.
4. We continue in this fashion order by order, until we find the first non-vanishing terms of order $i_{leading}$.
5. We add up all terms of this order. In this step the equivalence theorem can be of great benefit if applied carefully.

Contributions of Order 6

We quickly see that only the amplitude (5.11), which involves a pseudoscalar exchange, is of the order $m_{3/2}^{-1}$. The only terms in the amplitude's square $\propto m_{3/2}^{-6}$ come from the square of (5.11),

$$\begin{aligned}
\overline{|\mathcal{M}_{\text{PseudoScalar}}|^2} \propto& \frac{1}{m_{3/2}^2} \operatorname{Tr}\left[\Pi^{\nu\nu'}(k_1)\gamma^{\delta'}\Pi^{\mu'\mu}(k_2)\gamma^{\delta}\right] \\
=& \frac{1}{m_{3/2}^6} \operatorname{Tr}\left[\Pi^{\nu\nu'}_{(2)}(k_1)\gamma^{\delta'}\Pi^{\mu'\mu}_{(2)}(k_2)\gamma^{\delta}\right] \\
&+ \frac{1}{m_{3/2}^5}\left(\operatorname{Tr}\left[\Pi^{\nu\nu'}_{(2)}(k_1)\gamma^{\delta'}\Pi^{\mu'\mu}_{(1)}(k_2)\gamma^{\delta}\right] + \operatorname{Tr}\left[\Pi^{\nu\nu'}_{(1)}(k_1)\gamma^{\delta'}\Pi^{\mu'\mu}_{(2)}(k_2)\gamma^{\delta}\right]\right) \\
&+ \frac{1}{m_{3/2}^4} \operatorname{Tr}\left[\Pi^{\nu\nu'}_{(1)}(k_1)\gamma^{\delta'}\Pi^{\mu'\mu}_{(1)}(k_2)\gamma^{\delta}\right] + \mathcal{O}(m_{3/2}^{-3})
\end{aligned} \tag{5.14}$$

Now we will have to calculate the trace of many γ-matrices. For calculations like this we will use the Mathematica package FeynCalc[69, 70][2]. After the evaluation of these traces we find that the contributions $\propto m_{3/2}^{-6}$ vanish upon choosing a frame of reference and substituting the kinetic relations (D.11-D.14). Following our procedure we move to the next power.

Contributions of Order 5

We immediately see that the contributions $\propto m_{3/2}^{-5}$ in (5.14) vanish due to odd numbers of γ-matrices in the trace. But there are also interference terms of the same order,

[2]The used code can be found in the app F.

$$\begin{aligned}
\mathcal{M}^\dagger_{\text{PseudoScalar}}\mathcal{M}_{\text{Photino}} \propto{}& \frac{1}{m_{3/2}} \operatorname{Tr}\left[\Pi^{\nu\nu'}(k_1)\gamma^{\delta'}\Pi^{\mu'\mu}(k_2)\Theta^{(6,7)}_{\text{Photino}}\right] \\
={}& \frac{1}{m_{3/2}^5} \operatorname{Tr}\left[\Pi^{\nu\nu'}_{(2)}(k_1)\gamma^{\delta'}\Pi^{\mu'\mu}_{(2)}(k_2)\Theta^{(\cancel{6},7)}_{\text{Photino}}\right] \\
&+ \frac{1}{m_{3/2}^4}\Bigg(\operatorname{Tr}\left[\Pi^{\nu\nu'}_{(1)}(k_1)\gamma^{\delta'}\Pi^{\mu'\mu}_{(2)}(k_2)\Theta^{(6,\cancel{7})}_{\text{Photino}}\right] \\
&+ \operatorname{Tr}\left[\Pi^{\nu\nu'}_{(2)}(k_1)\gamma^{\delta'}\Pi^{\mu'\mu}_{(1)}(k_2)\Theta^{(6,\cancel{7})}_{\text{Photino}}\right]\Bigg) + \mathcal{O}(m_{3/2}^{-3}) ,
\end{aligned} \tag{5.15}$$

$$\begin{aligned}
\mathcal{M}^\dagger_{\text{PseudoScalar}}\mathcal{M}_{\text{Graviton}} \propto{}& \frac{1}{m_{3/2}} \operatorname{Tr}\left[\Pi^{\nu\nu'}(k_1)\gamma^{\delta'}\Pi^{\mu'\mu}(k_2)\Theta^{(0,3,5)}_{\text{Graviton}}\right] \\
={}& \frac{1}{m_{3/2}^5} \operatorname{Tr}\left[\Pi^{\nu\nu'}_{(2)}(k_1)\gamma^{\delta'}\Pi^{\mu'\mu}_{(2)}(k_2)\Theta^{(\cancel{0},3,5)}_{\text{Graviton}}\right] \\
&+ \frac{1}{m_{3/2}^4}\Bigg(\operatorname{Tr}\left[\Pi^{\nu\nu'}_{(1)}(k_1)\gamma^{\delta'}\Pi^{\mu'\mu}_{(2)}(k_2)\Theta^{(0,\cancel{3},\cancel{5})}_{\text{Graviton}}\right] \\
&+ \operatorname{Tr}\left[\Pi^{\nu\nu'}_{(2)}(k_1)\gamma^{\delta'}\Pi^{\mu'\mu}_{(1)}(k_2)\Theta^{(0,\cancel{3},\cancel{5})}_{\text{Graviton}}\right]\Bigg) + \mathcal{O}(m_{3/2}^{-3}) ,
\end{aligned} \tag{5.16}$$

$$\begin{aligned}
\mathcal{M}^\dagger_{\text{PseudoScalar}}\mathcal{M}_{\text{Scalar}} \propto{}& \frac{1}{m_{3/2}} \operatorname{Tr}\left[\Pi^{\nu\nu'}(k_1)\gamma^{\delta'}\Pi^{\mu'\mu}(k_2)\Theta^{(0)}_{\text{Scalar}}\right] \\
={}& \frac{1}{m_{3/2}^5} \operatorname{Tr}\left[\Pi^{\nu\nu'}_{(2)}(k_1)\gamma^{\delta'}\Pi^{\mu'\mu}_{(2)}(k_2)\Theta^{(\cancel{0})}_{\text{Scalar}}\right] \\
&+ \frac{1}{m_{3/2}^4}\Bigg(\operatorname{Tr}\left[\Pi^{\nu\nu'}_{(1)}(k_1)\gamma^{\delta'}\Pi^{\mu'\mu}_{(2)}(k_2)\Theta^{(0)}_{\text{Scalar}}\right] \\
&+ \operatorname{Tr}\left[\Pi^{\nu\nu'}_{(2)}(k_1)\gamma^{\delta'}\Pi^{\mu'\mu}_{(1)}(k_2)\Theta^{(0)}_{\text{Scalar}}\right]\Bigg) + \mathcal{O}(m_{3/2}^{-3}) ,
\end{aligned} \tag{5.17}$$

The terms that vanish due to an odd number of γ-matrices are indicated by a crossed-out index, e.g. $\Theta^{(0,\cancel{3},\cancel{5})}_{\text{Graviton}}$. For the remaining terms we calculate the traces using FeynCalc. The terms $\propto m_{3/2}^{-5}$ vanish after substituting (D.11-D.14) just as before. The only non-vanishing terms are of the order $m_{3/2}^{-4}$.

Contributions of Order 4

First of all, we notice that the amplitudes (5.8-5.10) do not depend on the gravitino mass. Apart from their interference terms with $\mathcal{M}_{\text{PseudoScalar}}$ it is safe to compute their contributions in the equivalence theorem limit. The result will be of the order $m_{3/2}^{-4}$. We calculate the square of the amplitudes, average over the incoming photon spins and sum over the outgoing gravitino spins. After the insertion of the kinematic relations (D.11)-(D.14) we obtain

$$\begin{aligned}
&\frac{1}{4}\overline{|i\mathcal{M}_{\text{Photino}} + i\mathcal{M}_{\text{Graviton}} + i\mathcal{M}_{\text{Scalar}}|^2} = \\
&\quad = \frac{\kappa^4 s^2 m_{\tilde{\gamma}}^4}{288 m_{3/2}^4} \frac{1}{\left(x^2\sin^2(\theta) + 4x + 4\right)^2} \\
&\quad \Bigg(128x + 16\sin^2(\theta)(\cos(2\theta) + 11)x^2 + 4(4\cos(2\theta) + 3\cos(4\theta) + 25)x^3 \\
&\qquad + \sin^2(\theta)(12\cos(2\theta) + \cos(4\theta) + 51)x^4 + 8\sin^4(\theta)x^5 \Bigg) + \mathcal{O}(m_{3/2}^{-2}) \,, \qquad (5.18)
\end{aligned}$$

where $x \equiv \frac{s}{m_{\tilde{\gamma}}^2}$. As discussed before the only non-vanishing contribtion from (5.14) is of the order $m_{3/2}^{-4}$,

$$\frac{1}{4}\overline{|\mathcal{M}_{\text{PseudoScalar}}|^2} = \frac{\kappa^4 s^2 m_{\tilde{\gamma}}^4}{36 m_{3/2}^4} x + \mathcal{O}(m_{3/2}^{-3}) \,. \qquad (5.19)$$

We turn to the interference terms in (5.15-5.17). Due to the fermion propagator the amplitudes (5.8) include terms with odd and even numbers of γ-matrices, so does (5.9). The only non-zero terms are given by

$$\begin{aligned}
&\frac{1}{4}\overline{\left(\mathcal{M}^{\dagger}_{\text{PseudoScalar}} \mathcal{M}_{\text{Photino}}\right)} + \text{c.c.} = \\
&\qquad - \frac{\kappa^4 s^2 m_{\tilde{\gamma}}^4}{18 m_{3/2}^4} \frac{\left(x^3\sin^2(\theta) + x^2\cos(2\theta) + 3x^2\right)}{x^2\sin^2(\theta) + 4x + 4} \,. \qquad (5.20)
\end{aligned}$$

Result for the Cross-Section

We add up (5.18), (5.19) and (5.20) to obtain the complete square of the overall amplitude. We substitute this into (D.23)[3]. After integration we are left with the

[3] We point out that we have to include an overall factor of $\frac{1}{2}$ coming from the Majorana nature of the gravitinos.

total cross-section of $\gamma\gamma \to \tilde{G}\tilde{G}$,

$$\sigma(\gamma\gamma \to \tilde{G}\tilde{G}) = \frac{\kappa^4 m_{\tilde{\gamma}}^4 s}{1728\pi m_{3/2}^4} \times \left[\frac{1}{1+x}\left(x + 7 - \frac{12}{x} - \frac{24}{x^2}\right) + \frac{1}{x+2}\left(\frac{48}{x^3} + \frac{24}{x^2} - \frac{6}{x}\right)\log(1+x)\right]. \quad (5.21)$$

This is in agreement with the results in [55, 60][4]. For $x \ll 1$, i.e. a large photino mass, this yields

$$\sigma(\gamma\gamma \to \tilde{g}\tilde{g}) = \frac{\kappa^4 s^2 m_{\tilde{\gamma}}^2}{576\pi m_{3/2}^4} + \mathcal{O}(x^0). \quad (5.22)$$

Heavy Sgoldstinos

Up until now we always assumed the sgoldstinos to be very light ($m_S, m_P \ll m_{\tilde{\gamma}}$). However many models contain very heavy sgoldstinos. The last two diagrams in (5.6) do not contribute significantly if $m_S, m_P \gg m_{\tilde{\gamma}}$ holds.
Without the s-channel exchange of the scalar and pseudoscalar field the cross-section for the gravitino pair production can be easily computed by using (4.18-4.19), since the critical $\mathcal{M}_{\text{PseudoScalar}}$ is no longer present. It is given by

$$\begin{aligned}\sigma(\gamma\gamma \to \tilde{G}\tilde{G}) &= \frac{\kappa^4 m_{\tilde{\gamma}}^4 s}{1728\pi m_{3/2}^4} \times \\ &\left[\frac{1}{1+x}\left(3x^2 - 8x - 5 - \frac{12}{x} - \frac{24}{x^2}\right) + \frac{6}{2+x}\left(2 + \frac{3}{x} + \frac{4}{x^2} + \frac{8}{x^3}\right)\log(1+x)\right] \\ &= \frac{s^3\kappa^4}{5760\pi m_{3/2}^4} + \mathcal{O}(x). \end{aligned} \quad (5.23)$$

[4] The authors of these papers however don't mention the necessity of retaining higher order terms in the gravitino polarization tensor.

5.2.2. Bounds from SN1987A

Case I – Superlight Sgoldstinos

Now we can substitute the cross-section (5.22) into (5.3) and integrate over the photons momenta using (5.4) and (5.5). We find

$$L_{\gamma\gamma,I} > \frac{160}{\pi^5}\left(\frac{\kappa}{m_{3/2}}\right)^4 m_{\tilde{\gamma}}^2 V T_{SN}^{11} \,. \tag{5.24}$$

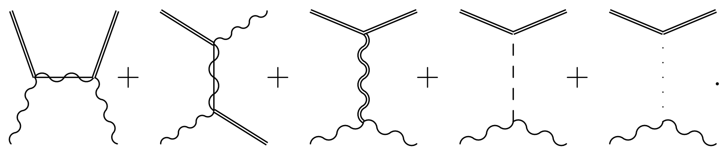
We re-arrange this inequality and apply the constraint on the missing energy (2.6) emitted from the Supernova core,

$$\begin{aligned} m_{3/2} &> \kappa\left(\frac{160}{\pi^5}\right)^{1/4}\left(\frac{V}{10^{52}\mathrm{erg/s}}\right)^{1/4} m_{\tilde{\gamma}}^{1/2} T_{SN}^{11/4} \\ &= 1.8\times 10^{-5}\left(\frac{m_{\tilde{\gamma}}}{100\mathrm{GeV}}\right)^{1/2}\left(\frac{T_{SN}}{0.05\mathrm{GeV}}\right)^{11/4}\left(\frac{V}{4.2\times 10^{18}\mathrm{cm}^3}\right)^{1/4} \mathrm{eV}\,. \end{aligned} \tag{5.25}$$

Here we used the conversion factors from app. A.
This constraint is valid under the condition that the gravitinos, once produced, leave the SN core without further interactions. This means that the gravitino's mean-free-path λ must exceed the Supernova core radius R_{SN}. The gravitinos scatter mainly with photons [1], i.e. $\gamma\tilde{G} \longrightarrow \gamma\tilde{G}$. Their mean-free-path is given by

$$\lambda_{\mathrm{MFP}} \sim \left(n_\gamma(T_{SN})\sigma(\gamma\tilde{G} \longrightarrow \gamma\tilde{G})\right)^{-1} \,. \tag{5.26}$$

The contributing diagrams to gravitino-photon scattering are given by

$i\mathcal{M} =$  . (5.27)

We calculate the cross-section as we did in sec. 5.2.1 using FeynCalc and obtain

$$\sigma(\gamma\tilde{G} \longrightarrow \gamma\tilde{G}) = \frac{\kappa^4 m_{\tilde{\gamma}}^2 s^2}{2304\pi m_{3/2}^4} + \mathcal{O}(x^0)\,. \tag{5.28}$$

In evaluating (5.26) we need the number density and the average energy of photons. The corresponding relations can be found in app. E, namely (E.6)-(E.8). We obtain the gravitino mean-free-path in the SN core,

$$\lambda_{\text{MFP}} \sim \frac{8\pi^3}{9\zeta(3)} m_{3/2}^4 \kappa^{-4} m_{\tilde{\gamma}}^{-2}\, T^{-7}\,. \tag{5.29}$$

The bound we found in (5.25) is only valid if $\lambda_{\text{MFP}} > R_{SN}$. Plugging in numbers we find

$$\begin{aligned}\lambda_{\text{MFP}} &\sim 2.1 \times 10^6 \left(\frac{m_{3/2}}{1.8\times 10^{-5}\text{eV}}\right)^4 \left(\frac{m_{\tilde{\gamma}}}{100\text{GeV}}\right)^{-2} \left(\frac{T}{0.05\text{GeV}}\right)^{-7} \text{km} \\ &\gg 10 \left(\frac{R_{SN}}{10\text{km}}\right) \text{km}\,.\end{aligned} \tag{5.30}$$

For our allowed range for $m_{3/2}$ the gravitinos leave the supernova core without scattering, our bounds are reasonable.
Yet looking at (5.30) we quickly recognize that for even smaller values of $m_{3/2}$ its mean-free-path gets short enough for the gravitinos to diffuse inside the core. If the gravitinos are trapped inside the core for longer than 1s energy is depleted by neutrino emission and the gravitinos' luminosity is lower and again compatible with $L_X < 10^{52}\frac{\text{erg}}{\text{s}}$. Trapped gravitinos random-walk through the core and move a distance $R_{\text{diff}} \sim \sqrt{N}\lambda_{\text{mfp}}$ in time interval $t_{\text{diff}} = \frac{\lambda_{\text{mfp}}}{c}N$, where N is the number of scatterings. Hence the conditions for the decreased luminosity are

$$\begin{aligned}&\frac{\lambda_{\text{mfp}}}{c}N \geq 1\text{s}\,, \quad \lambda_{\text{mfp}}\sqrt{N} \leq R_{SN} \\ \Longrightarrow\ &\lambda_{\text{mfp}} \leq \frac{R_{SN}^2}{c(1\text{s})} \approx 0.3\text{m}\,.\end{aligned} \tag{5.31}$$

Substitution of (5.29) yields

$$m_{3/2} \leq 6.2 \times 10^{-8}\text{eV} \tag{5.32}$$

This is the allowed mass range for the gravitino in the case of gravitino diffusion. As a final result we can exclude the mass range of

$$6.2 \times 10^{-8}\mathrm{eV} < m_{3/2} < 1.8 \times 10^{-5}\mathrm{eV}\,, \tag{5.33}$$

based on the observation of the SN1987A supernova. This bound heavily relies on the sgoldstinos being light.

Case II – Heavy Sgoldstinos

We integrate (5.3) using the cross-section (5.23) just as we did in the case of massless sgoldstinos and get

$$L_{\gamma\gamma,II} > \frac{1536}{\pi^5}\left(\frac{\kappa}{m_{3/2}}\right)^4 V T_{SN}^{13}\,, \tag{5.34}$$

leading to the weaker bounds of

$$m_{3/2} > 7.0 \times 10^{-7}\left(\frac{T_{SN}}{0.05\mathrm{GeV}}\right)^{13/4}\left(\frac{V}{4.2\times 10^{18}\mathrm{cm}^3}\right)^{1/4}\mathrm{eV}\,. \tag{5.35}$$

In the case of very heavy Sgoldstinos, the last two diagrams in (5.27) do not contribute significantly and can be neglected. This changes our scattering-cross-section to

$$\begin{aligned}\sigma(\gamma\tilde{G}\longrightarrow\gamma\tilde{G}) = {} & \frac{\kappa^4 m_{\tilde{\gamma}}^4 s}{3456\pi m_{3/2}^4}\times \\ & \left[\frac{1}{2(x-1)^2(1+x)}\left(24x^5+3x^4+x^3+25x^2+41x-52-\frac{78}{x}+\frac{60}{x^2}\right)\right. \\ & \left. -\frac{6}{x-1}\left(2+\frac{3}{x}+\frac{4}{x^2}-\frac{5}{x^2}\right)\log(1+x)\right] \\ = {} & \frac{s^3\kappa^4}{768\pi m_{3/2}^4}+\mathcal{O}(x)\,. \end{aligned} \tag{5.36}$$

The mean-free-path becomes

$$\lambda_{\text{MFP}} \sim \frac{2\pi^3}{243\zeta(3)} m_{3/2}^4 \kappa^{-4}\, T^{-9} \tag{5.37}$$

$$= 1.8 \times 10^5 \left(\frac{m_{3/2}}{7.0 \times 10^{-7}\text{eV}}\right)^4 \left(\frac{T}{0.05\text{GeV}}\right)^{-9} \text{km} \gg 10 \left(\frac{R_{SN}}{10\text{km}}\right) \text{km}\,. \tag{5.38}$$

The obtained bound (5.35) is consistent.
Again for sufficiently small values of $m_{3/2}$ the gravitinos will diffuse in the core leading to decreased luminosity that spoils this bound. Substituting (5.37) into (5.31), we find that this is the case for

$$m_{3/2} < 4.5 \times 10^{-9}\text{eV}\,. \tag{5.39}$$

In conclusion, for heavy sgoldstinos, the observation of SN1987A allows us to exclude the mass range

$$4.5 \times 10^{-9}\text{eV} < m_{3/2} < 7.0 \times 10^{-7}\text{eV}\,. \tag{5.40}$$

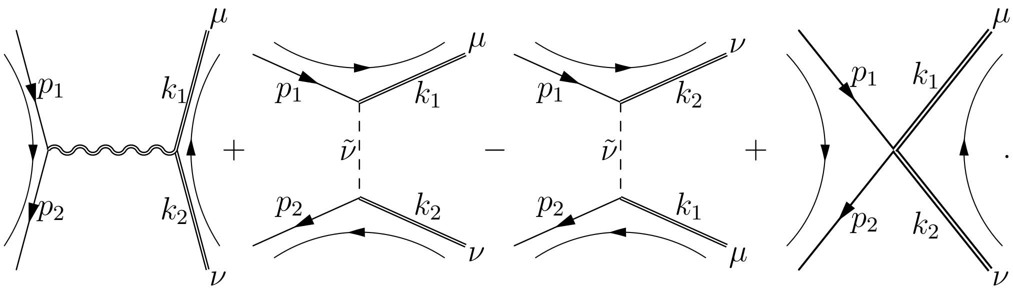
This bound is our most conservative one from the consideration of photons. All of the relevant couplings are contained in any SUGRA theory and we did not assume to have any additional couplings rising from the hidden field sector. Yet, the constraint can be tightened by additional gravitino production channels as we will show now.

5.3. Gravitino Pair Production via Neutrino Collision

We will now perform a similar analysis using the channel $\nu(p_1)\overline{\nu}(p_2) \longrightarrow \tilde{G}(k_1)\tilde{G}(k_2)$.

5.3.1. Calculation of the Cross-Section

The three contributing diagrams are given by

$i\mathcal{M} =$  .

With the Feynman rules from sec. 4.3 and app. C we find the amplitudes,

$$
\begin{aligned}
i\mathcal{M}_1 = &-\frac{\kappa^2}{8(p_1+p_2)^2}\overline{v}(p_2)\left[\gamma^\alpha(p_1-p_2)^\beta + \gamma^\beta(p_1-p_2)^\alpha - \eta^{\alpha\beta}(\not{p}_2 - \not{p}_1)\right]P_L u(p_1) \\
&\times \overline{\psi}^{+\,\mu}(k_1)\left[\epsilon_{\mu\lambda\nu\alpha}\gamma^5\gamma_\beta(k_2-k_1)^\lambda + \frac{i}{2}\epsilon_{\mu\lambda\nu\alpha}\gamma^5\left\{\gamma^\lambda,\sigma_{\beta\tau}\right\}(p_1+p_2)^\tau\right]\psi^{-\,\nu}(k_2) \\
\approx &-\frac{\kappa^2}{12m_{3/2}^2}\frac{k_1^\mu k_2^\nu}{(p_1+p_2)^2} \\
&\times \overline{v}(p_2)\left[\gamma^\alpha(p_1-p_2)^\beta + \gamma^\beta(p_1-p_2)^\alpha - \eta^{\alpha\beta}(\not{p}_2 - \not{p}_1)\right]P_L u(p_1) \\
&\times \overline{u}(k_1)\left[\epsilon_{\mu\lambda\nu\alpha}\gamma^5\gamma_\beta(k_2-k_1)^\lambda + \frac{i}{2}\epsilon_{\mu\lambda\nu\alpha}\gamma^5\left\{\gamma^\lambda,\sigma_{\beta\tau}\right\}(p_1+p_2)^\tau\right]v(k_2)\,, \quad (5.41) \\
i\mathcal{M}_2 = &\frac{i\kappa^2}{2}\frac{1}{(p_1-k_1)^2 - m_{\tilde{\nu}}^2} \\
&\times \overline{\psi}^{+\,\mu}(k_1)(\not{p}_1 - \not{k}_1)\gamma_\mu P_L u(p_1)\,\overline{v}(p_2)P_R\gamma_\nu(\not{k}_2 - \not{p}_2)\psi^{-\,\nu}(k_2) \\
\approx &\frac{i\kappa^2}{3m_{3/2}^2}\frac{1}{(p_1-k_1)^2 - m_{\tilde{\nu}}^2} \\
&\times \overline{u}(k_1)(\not{p}_1 - \not{k}_1)\not{k}_1 P_L u(p_1)\,\overline{v}(p_2)P_R\not{k}_2(\not{k}_2 - \not{p}_2)\psi^\nu(k_2) \\
\approx &-\frac{i\kappa^2}{3m_{3/2}^2\left((p_1-k_1)^2 - m_{\tilde{\nu}}^2\right)}\,\overline{u}(k_1)\not{p}_1\not{k}_1 P_L u(p_1)\,\overline{v}(p_2)P_R\not{k}_2\not{p}_2 v(k_2)\,, \qquad (5.42) \\
i\mathcal{M}_3 \approx &-\frac{i\kappa^2}{3m_{3/2}^2\left((p_1-k_2)^2 - m_{\tilde{\nu}}^2\right)}\,\overline{u}(k_2)\not{p}_1\not{k}_2 P_L u(p_1)\,\overline{v}(p_2)P_R\not{k}_1\not{p}_2 v(k_1)\,, \qquad (5.43)
\end{aligned}
$$

$$
\begin{aligned}
i\mathcal{M}_4 &= \frac{\kappa^2}{4}\overline{\psi}^{+\mu}(k_1)\left[\epsilon_{\mu\lambda\nu\kappa}\gamma^\lambda P_R - i\eta_{\mu\nu}\gamma_\kappa P_R\right]\psi^{-\nu}(k_2)\,\overline{v}(p_2)\gamma^\kappa P_L u(p_1) \\
&\approx \frac{\kappa^2}{6m_{3/2}^2}k_1^\mu k_2^\nu \overline{u}(k_1)\left[\epsilon_{\mu\lambda\nu\kappa}\gamma^\lambda P_R - i\eta_{\mu\nu}\gamma_\kappa P_R\right]v(k_2)\,\overline{v}(p_2)\gamma^\kappa P_L u(p_1)\,. \qquad (5.44)
\end{aligned}
$$

The last amplitude of the diagram with the 4-fermion vertex can be read off the last line of our Lagrangian in (4.13). For this process it is safe to use the equivalence theorem from the beginning and continue with the massless goldstino instead of the gravitino. The leading order is given by $\frac{\kappa^2}{m_{3/2}^2}$. In this limit we already made use of $\not{k}_i\not{k}_i = k_i^2 \approx 0$ for the amplitudes (5.42) and (5.43).
Interference terms including the third amplitude like $\mathcal{M}_3^\dagger\mathcal{M}_2$ will cause problems because they will not lead to calculable γ traces without further ado. The reason for this is the Majorana nature of the gravitino. However, it is possible to transform the interference terms in such a way that we can just calculate the spin sums as usual using FeynCalc [71]. For this we need the relations (B.13), (B.14) and (B.17). For clarity we will demonstrate the necessary steps on an example,

$$
\begin{aligned}
&\overline{v}(k)\not{p}\not{k}P_R v(p) \overset{(B.14)}{=} \overline{v}(k)\not{p}\not{k}P_R C\overline{u}^T(p) = \left(\overline{v}(k)\not{p}\not{k}P_R C\overline{u}^T(p)\right)^T \\
&= \overline{u}(p)C^T P_R^T \not{k}^T \not{p}^T (v(k)^\dagger\gamma^0)^T \overset{(B.13)}{=} \overline{u}(p)CC^{-1}P_R CC^{-1}\not{k}CC^{-1}\not{p}CC^{-1}\gamma^0 C v^*(k) \\
&\overset{(B.17)}{=} \overline{u}(p)P_R\not{k}\not{p}u(k)\,.
\end{aligned}
$$

This way we can write the problematic interference terms as

$$
\begin{aligned}
\mathcal{M}_3^\dagger\mathcal{M}_1 = &\frac{-i\kappa^4 k_1^\mu k_2^\nu}{36m_{3/2}^4(p_1+p_2)^2((p_1-k_2)^2 - m_{\tilde{\nu}}^2)}\overline{u}(p_1)P_L\not{k}_2\not{p}_1 u(k_2) \\
&\times \overline{v}(k_1)\not{p}_2\not{k}_1 P_R v(p_2) \\
&\times \overline{v}(p_2)\left(\gamma^\alpha(p_2-p_1)^\beta + \gamma^\beta(p_2-p_1)^\alpha - 3\eta^{\alpha\beta}(\not{p}_2 - \not{p}_1)\right)u(p_1) \\
&\times \overline{u}(k_1)\left(\epsilon_{\mu\lambda\nu\alpha}\gamma^5\gamma_\beta(k_2-k_1)^\lambda + \frac{i}{2}\epsilon_{\mu\lambda\nu\alpha}\gamma^5\left\{\gamma^\lambda,\sigma_{\beta\tau}\right\}(p_1+p_2)^\tau\right)v(k_2)
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{-i\kappa^4 k_1^\mu k_2^\nu}{36m_{3/2}^4(p_1+p_2)^2((p_1-k_2)^2-m_{\tilde{\nu}}^2)}\overline{u}(p_1)P_L\not{k}_2\not{p}_1 u(k_2) \\
&\times \overline{u}(k_2)\left(\epsilon_{\mu\lambda\nu\alpha}\gamma_\beta\gamma^5(k_1-k_2)^\lambda + \frac{i}{2}\epsilon_{\mu\lambda\nu\alpha}\left\{\gamma^\lambda,\sigma_{\beta\tau}\right\}\gamma^5(p_1+p_2)^\tau\right)v(k_1) \\
&\times \overline{v}(k_1)\not{p}_2\not{k}_1 P_R v(p_2) \\
&\times \overline{v}(p_2)\left(\gamma^\alpha(p_1-p_2)^\beta + \gamma^\beta(p_1-p_2)^\alpha - \eta^{\alpha\beta}(\not{p}_2-\not{p}_1)\right)u(p_1)\,, \\
\mathcal{M}_3^\dagger\mathcal{M}_2 &= \frac{\kappa^4}{9m_{3/2}^4\left((p_1-k_1)^2-m_{\tilde{\nu}}^2\right)\left((p_1-k_2)^2-m_{\tilde{\nu}}^2\right)}\overline{u}(p_1)P_L\not{k}_2\not{p}_1 u(k_2) \\
&\times \overline{v}(k_1)\not{p}_2\not{k}_1 P_R v(p_2) \quad \overline{u}(k_1)\not{p}_1\not{k}_1 P_L u(p_1) \quad \overline{v}(p_2)P_R\not{k}_2\not{p}_2 v(k_2) \\
&= \frac{\kappa^4}{9m_{3/2}^4\left((p_1-k_1)^2-m_{\tilde{\nu}}^2\right)\left((p_1-k_2)^2-m_{\tilde{\nu}}^2\right)}\overline{v}(k_2)\not{p}_1\not{k}_2 P_L v(p_1) \\
&\times \overline{v}(p_1)P_L\not{k}_1\not{p}_1 v(k_1) \quad \overline{v}(k_1)\not{p}_2\not{k}_1 P_R v(p_2) \quad \overline{v}(p_2)P_R\not{k}_2\not{p}_2 v(k_2)\,, \\
\mathcal{M}_3^\dagger\mathcal{M}_4 &= \frac{i\kappa^4 k_1^\mu k_2^\nu}{18m_{3/2}^4((p_1-k_2)^2-m_{\tilde{\nu}}^2)}\overline{u}(p_1)P_L\not{k}_2\not{p}_1 u(k_2) \quad \overline{v}(k_1)\not{p}_2\not{k}_1 P_R v(p_2) \\
&\times \overline{v}(p_2)\gamma^\kappa P_L u(p_1) \quad \overline{u}(k_1)\left(\epsilon_{\mu\lambda\nu\kappa}\gamma^\lambda P_R - i\eta_{\mu\nu}\gamma_\kappa P_R\right)v(k_2) \\
&= \frac{i\kappa^4 k_1^\mu k_2^\nu}{18m_{3/2}^4((p_1-k_2)^2-m_{\tilde{\nu}}^2)}\overline{v}(k_1)\not{p}_2\not{k}_1 P_R v(p_2) \quad \overline{v}(p_2)\gamma^\kappa P_L u(p_1) \\
&\times \overline{u}(p_1)P_L\not{k}_2\not{p}_1 u(k_2) \quad \overline{u}(k_2)\left(i\eta_{\mu\nu}P_R\gamma_\kappa - \epsilon_{\mu\lambda\nu\kappa}P_R\gamma^\lambda\right)v(k_1)\,.
\end{aligned}
$$

We can now average over initial and sum over final spins as usual and find

$$
\begin{aligned}
-\frac{1}{4}\sum_{\text{spins}}&\left(\mathcal{M}_3^\dagger\mathcal{M}_1 + \mathcal{M}_3^\dagger\mathcal{M}_2 + \mathcal{M}_3^\dagger\mathcal{M}_4 + \text{c.c.}\right) = \\
&\qquad\qquad \frac{\kappa^4 s^5(1+\cos(\theta))^4(1-2\cos(\theta))}{576m_{3/2}^4\left(2m_s^2+s\cos(\theta)+s\right)}\,. \qquad (5.45)
\end{aligned}
$$

Here we already chose a frame of reference by using (D.15) and (D.16). The other contributions are calculated as usual using FeynCalc. We add up the contributions, integrate and obtain the total cross-section for gravitino production via neutrino collision,

$$\sigma(\nu\overline{\nu} \longrightarrow \tilde{G}\tilde{G}) = \frac{\kappa^4 s^3}{32256\pi m_{3/2}^4}\left(\frac{1}{1+x}\left(4x^3 + 4x^2 + 7x^1 - 7 + 14x^{-1} - 28x^{-2} + 84x^{-3} + 168x^{-4}\right) - \frac{168}{x^5}\log(1+x)\right). \tag{5.46}$$

Here we defined the quantity $x \equiv \frac{s}{m_{\tilde{\nu}}^2}$. For $x \ll 1$ we find

$$\sigma(\nu\overline{\nu} \longrightarrow \tilde{G}\tilde{G}) = \frac{\kappa^4 s^3}{23040\pi m_{3/2}^4} + \mathcal{O}(x^2)\,. \tag{5.47}$$

5.3.2. Bounds from SN1987A

Our next step is to calculate the gravitino luminosity in a supernova core. In contrast to (5.2) we have to use the Fermi-Dirac distribution function n_ν according to the neutrinos fermionic nature,

$$L = 3 \times \frac{8V}{(2\pi)^6}\int \mathrm{d}^3p_1 n_\nu(p_1^0)\int \mathrm{d}^3p_2 n_\nu(p_2^0)(p_1^0 + p_2^0)\frac{p_1 \cdot p_2}{p_1^0 p_2^0}\sigma(\nu\overline{\nu} \to \tilde{G}\tilde{G})\,. \tag{5.48}$$

The factor of 3 takes the three different flavors of neutrinos into account, whose superpartners we assume to be degenerate in mass for simplicity.
Apart from this distinction we proceed in the same way as in sec. 5.1 and integrate over the momenta using spherical coordinates. The luminosity is given by

$$L_{\nu\overline{\nu}} = \frac{93\zeta(7)\pi}{80}\frac{\kappa^4 V T_{SN}^{13}}{m_{3/2}^4}\,, \quad \text{where } \zeta(7) \approx 1\,. \tag{5.49}$$

Comparison with Former Results We compare our result for the gravitino luminosity with our results for photons in the initial state. In the case of superlight sgoldstinos we obtained a large luminosity (see (5.24)) compared to (5.49). More precisely the ratio

$$\frac{L_{\nu\overline{\nu}}}{L_{\gamma\gamma,I}} \approx 2 \times 10^{-6}\left(\frac{T_{SN}}{50\mathrm{MeV}}\right)^2\left(\frac{m_{\tilde{\gamma}}}{100\mathrm{GeV}}\right)^{-2} \tag{5.50}$$

clearly illustrates that the channel $\nu\overline{\nu} \to \tilde{G}\tilde{G}$ does not contribute significantly to the gravitino production, because the production via photon collision dominates via the exchange of sgoldstinos. In our model there are no similar enhanced couplings for the neutrinos.

Nevertheless we also considered the other alternative, where the sgoldstino contributions are suppressed by their heavy mass. The $\gamma\gamma \to \tilde{G}\tilde{G}$ does not dominate the other channels anymore, as we see by comparing (5.34) with (5.49),

$$\frac{L_{\nu\overline{\nu}}}{L_{\gamma\gamma,II}} \approx 0.7 \,. \tag{5.51}$$

The bounds derived in the second part of sec. 5.2.2 therefore become a little bit more restrictive. We add up the luminosities of the two production channels,

$$L_{\text{total}} \equiv L_{\gamma\gamma,II} + L_{\nu\overline{\nu}} \,, \tag{5.52}$$

and demand that $L_{\text{total}} < L_X$, see (2.6).

$$m_{3/2} > 8.0 \times 10^{-7} \left(\frac{T_{SN}}{50\text{MeV}}\right)^{13/4} \left(\frac{V}{4.2 \times 10^{18}\text{cm}^3}\right)^{1/4} \text{eV} \,. \tag{5.53}$$

The scattering of gravitinos on neutrinos might also be relevant in the context of the core's opacity. The cross-section $\sigma(\nu\tilde{G} \to \nu\tilde{G})$ is very similar to the one computed above and we obtain

$$\sigma(\nu\tilde{G} \to \nu\tilde{G}) = \frac{13\kappa^4 s^3}{10240\pi m_{3/2}^4} \,. \tag{5.54}$$

The ratio $\frac{\sigma(\nu\tilde{G}\to\nu\tilde{G})}{\sigma(\gamma\tilde{G}\to\gamma\tilde{G})} \approx 0.98$ shows that this is a relevant gravitino scattering process. The mean-free-path gets modified,

$$\begin{aligned} \lambda_{\text{MFP}} &= \Big(n_\gamma(T)\sigma(\gamma\tilde{G} \to \gamma\tilde{G}) + 3 \times n_\nu(T)\sigma(\nu\tilde{G} \to \nu\tilde{G})\Big)^{-1} \\ &\approx \frac{320\pi^3}{124173\zeta(3)} m_{3/2}^4 \kappa^{-4}\, T^{-9} && (5.55) \\ &= 9.7 \times 10^4 \left(\frac{m_{3/2}}{8.0 \times 10^{-7}\text{eV}}\right)^4 \left(\frac{T}{0.05\text{GeV}}\right)^{-9} \text{km} \gg 10 \left(\frac{R_{SN}}{10\text{km}}\right) \text{km} \,. && (5.56) \end{aligned}$$

By including the neutrinos into our analysis the mean-free-path shrunk compared to (5.37). However the decreased value still allows the gravitinos to escape the SN core as long as (5.31) does not hold. In other words, our argument is consistent only if

$$m_{3/2} > 6.0 \times 10^{-9}\mathrm{km}\,. \tag{5.57}$$

In conclusion, the additional consideration of neutrinos as the initial state allows us to tighten the excluded intervall for the gravitino mass in the case of heavy scalars,

$$6.0 \times 10^{-9}\mathrm{eV} < m_{3/2} < 8.0 \times 10^{-7}\mathrm{eV}\,. \tag{5.58}$$

The gravitino production rate and the gravitino scattering cross-sections with photons and neutrinos are of same order. Their combination therefore modifies the excluded mass range only slightly.

6

Phenomenological Implications of Bilinear RPV on Supernova Bounds

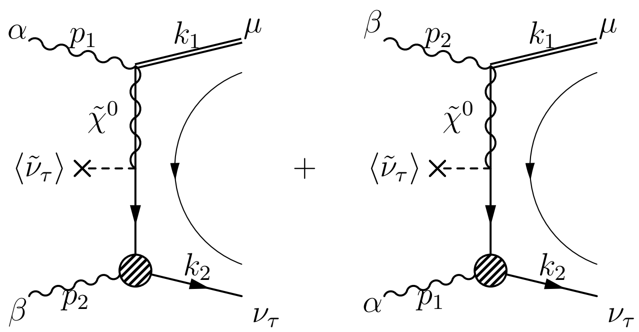
In the second part we allow certain additional terms in our superpotential that violate R-parity. As discussed in ch. 4 new effective couplings emerge and the production of single gravitinos is feasible.

For both initial states from ch. 5 we show that the production rates due to new RPV channels are negligible. Therefore the observation of the SN neutrinos of SN1987A does not allow us to make any statement about bilinear R-parity violations.

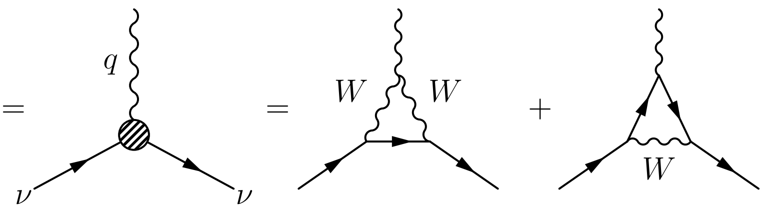
As we saw in sec. 4.4, the inclusion of bilinear R-parity violation gives rise to effective vertices between gravitino, neutrino and photons or Z bosons. In this way we can not only expect gravitino pair production from photon collision that we computed earlier, but also the production of single gravitinos.

6.1. Single Gravitino Production via Photon Collision

We investigate whether the channel $\gamma\gamma \to \tilde{G}\nu$ is relevant compared to $\gamma\gamma \to \tilde{G}\tilde{G}$. The two contributing diagrams are given by

$i\mathcal{M} =$  (6.1)

The 'blob' in this diagram denotes the coupling of the neutrinos to the photon field. Since the neutrino is neutral this vertex is not present at tree level of course. However it could be generated by radiative corrections, such as

$V_\mu^{\gamma\nu\nu} =$  $+ \cdots .$ (6.2)

Depending on the model there may be many more contributing diagrams. Instead of focusing on this potentially large number of contributing loop diagrams we take a general vertex with form factors [72],

$$V_\mu^{\gamma\nu\nu}(q^2) = f_Q(q^2)\gamma_\mu - f_M(q^2)i\sigma_{\mu\nu}q^\nu + f_E(q^2)\sigma_{\mu\nu}q^\nu\gamma_5 + f_A(q^2)\left(q^2\gamma_\mu - q_\mu \not{q}\right)\gamma_5 \,.$$

The occuring functions are the real charge, magnetic dipole, electric dipole and anapole neutrino form factors respectively. For the coupling with real photons, i.e. $q^2 = 0$, the quantities

$$f_Q(0) = q_\nu \,, \quad f_M(0) = \mu_\nu \,, \quad f_E(0) = e_\nu \,, \text{ and } f_A(0) = a_\nu$$

are the effective charge, dipoles and anapole of a neutrino. These are strictly limited by observations [36],

$$\begin{aligned} q_\nu &< \times 10^{-12} e \approx 10^{-13} \,, \\ \mu_\nu, e_\nu &< 10^{-7}\mu_B \approx 10^{-5}\mathrm{GeV}^{-1} \,, \\ a_\nu &= \frac{r_\nu^2}{6} < 10^{-6}\mathrm{GeV}^{-2} \,, \end{aligned}$$

here r_ν^2 is the 'neutrino charge radius' squared.
With the vertex from (4.20) we find the amplitude

$$\begin{aligned} & i\mathcal{M}(\gamma\gamma \longrightarrow \tilde{G}\nu) = \\ & \quad -\frac{\kappa g_Z \langle\tilde{\nu}\rangle U_{\tilde{\gamma}\tilde{Z}}}{8\sqrt{2}m_Z} \epsilon_1^\alpha \epsilon_2^\beta \overline{u}(k_2) V_\beta^{\gamma\nu\nu}(p_2^2) \frac{\not{p}_1 - \not{k}_1 + m_\nu}{(p_1 - k_1)^2 - m_\nu^2} (1+\gamma_5)\gamma_\mu \left[\not{p}_1, \gamma_\alpha\right] \psi^\mu(k_1) \\ & \quad + ((p_1, \alpha) \leftrightarrow (p_2, \beta)) \,. \end{aligned} \tag{6.3}$$

For the total cross-section, we find

$$\begin{aligned} \sigma(\gamma\gamma \longrightarrow \tilde{G}\nu) &= \frac{\kappa^2 U_{\tilde{\gamma}\tilde{Z}}^2 \langle\tilde{\nu}\rangle^2 g_Z^2 \; s}{576\pi m_{3/2}^2 m_Z^2} \left(3s\left(f_E^2(0) + f_M^2(0)\right) - 5f_Q(0)^2 - 2f_A(0)f_Q(0)s\right) \\ &= \frac{\kappa^2 U_{\tilde{\gamma}\tilde{Z}}^2 \langle\tilde{\nu}\rangle^2 g_Z^2 \; s}{576\pi m_{3/2}^2 m_Z^2} \left(3s\left(\epsilon_\nu^2 + \mu_\nu^2\right) - 5q_\nu^2 - 2a_\nu q_\nu s\right) \,. \end{aligned} \tag{6.4}$$

Now we can determine whether this production rate is of any relevance compared to the production of gravitino pairs from the same initial state. We consider the ratio of (5.23) with (6.4).

$$\frac{\sigma(\gamma\gamma \longrightarrow \tilde{G}\nu)}{\sigma(\gamma\gamma \rightarrow \tilde{G}\tilde{G})} = \frac{10 g_Z^2 U_{\tilde{\gamma}\tilde{Z}}^2 \langle\tilde{\nu}\rangle^2 \; m_{3/2}^2}{\kappa^2 s^2 m_Z^2} \left(3s\left(\epsilon_\nu^2 + \mu_\nu^2\right) - 5q_\nu^2 - 2a_\nu q_\nu s\right) \,. \tag{6.5}$$

We insert the numerical values for the RPV parameters from sec. 4.4 and the empirical bounds on the neutrino's electromagnetic properties and find

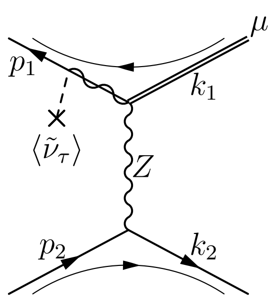
$$\frac{\sigma(\gamma\gamma \longrightarrow \tilde{G}\nu)}{\sigma(\gamma\gamma \to \tilde{G}\tilde{G})} < 8 \times 10^{-15} \left(\frac{m_{3/2}}{10^{-6}\text{eV}}\right)^2 \left(\frac{\xi}{10^{-7}}\right)^2 , \tag{6.6}$$

where we used $s = 36T_{SN}^2$ and $T_{SN} \approx 50$MeV.
Obviously the production of gravitinos via photon collisions occurs almost exclusively in pairs and bilinear RPV does not alter any of our previous results. Neither modifications of the luminosity nor the mean-free-path can be of relevance making any further calculation with this result unnecessary.
For heavier gravitinos ($m_{3/2} \sim \mathcal{O}(10\text{eV})$) the two processes may lead to similar production rates but then the absolute production rate would be completely negligible.

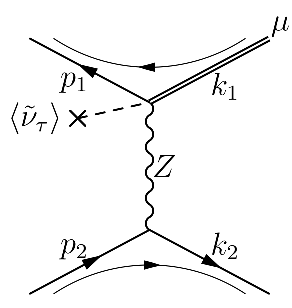
6.2. Single Gravitino Production via Neutrino Collision

If the sgoldstinos are very heavy, gravitino pair production from photon collision is not dominant and contributions from neutrino collisions are equally relevant for the gravitino luminosity. As in the case of photons, bilinear RPVs allow us to produce a single gravitino from the initial state $\nu\overline{\nu}$. We show that the luminosity from single gravitino production may safely be neglected.
The diagrams associated with the process $\nu(p_2)\overline{\nu}(p_1) \longrightarrow \tilde{G}(k_1)\nu(k_2)$ are

$i\mathcal{M} =$



$+$



.

They correspond to the following amplitudes,

$$
\begin{aligned}
i\mathcal{M}_1 = &-\frac{\kappa g_Z^2 U_{\tilde{Z}\tilde{Z}} \langle \tilde{\nu}_\tau \rangle}{8\sqrt{3} m_Z m_{3/2}} \frac{1}{(p_1 - k_1)^2 - m_Z^2} \\
&\overline{u}(k_2)\gamma^\alpha P_L u(p_2) \quad \overline{v}(p_1)(1+\gamma^5)\not{k}_1 \left[(\not{p}_1 - \not{k}_1), \gamma_\alpha\right] v(k_1)\,, \qquad (6.7) \\
i\mathcal{M}_2 = &-\frac{\kappa g_Z^2 \langle \tilde{\nu}_\tau \rangle}{4\sqrt{3} m_{3/2}} \frac{1}{(p_1 - k_1)^2 - m_Z^2} \\
&\overline{u}(k_2)\gamma^\alpha P_L u(p_2) \quad \overline{v}(p_1)(1+\gamma^5)\not{k}_1 \gamma_\alpha v(k_1)\,. \qquad (6.8)
\end{aligned}
$$

We compute the cross-section using FeynCalc,

$$
\begin{aligned}
\sigma(\nu\overline{\nu} \longrightarrow \tilde{G}\nu) &= \frac{\kappa^2 g_Z^4 \langle \tilde{\nu}_\tau \rangle^2}{576\pi m_{3/2}^2} \Bigg[U_{\tilde{Z}\tilde{Z}}^2 \left(y + 12 \left(1 + \frac{1}{y} \right) \right) - 6 \\
&\qquad - 3(2+y) \Big(2U_{\tilde{Z}\tilde{Z}^2}^2 (1+y) - y \Big) \log(1+y) \Bigg] \\
&= \frac{\kappa^2 g_Z^4 \langle \tilde{\nu}_\tau \rangle^2 s^2}{1152\pi m_{3/2}^2 m_Z^4} + \mathcal{O}(y^3)\,, \quad \text{where } y \equiv \frac{s}{m_Z^2} \ll 1\,. \qquad (6.9)
\end{aligned}
$$

Just as in (6.6) we compare this result with the corresponding cross-section from the R-parity conserving case (5.47),

$$
\frac{\sigma(\nu\overline{\nu} \longrightarrow \tilde{G}\nu)}{\sigma(\nu\overline{\nu} \to \tilde{G}\tilde{G})} \approx 4 \times 10^{-7} \left(\frac{m_{3/2}}{10^{-6}\text{eV}} \right)^2 \left(\frac{\xi}{10^{-7}} \right)^2 . \qquad (6.10)
$$

We come to the same conclusion as sec. 6.1. The extra gravitino production from neutrino-antineutrino collisions due to bilinear RPV is vanishingly low and has no influence on our results from ch. 5.

7 Conclusions

We started this thesis by covering both the astrophysical context, where we discussed supernovae and the energy-loss argument, and the current stand of particle physics. There we emphasized that particle physics phenomenology is of great relevance now more than ever, especially if it comes to physics beyond the SM. We introduced the idea of SUSY and SUGRA and presented the full Lagrangian for a locally supersymmetric gauge field theory. After a brief treatment of R-parity and bilinear R-parity violations we focused on the phenomenology of superlight gravitinos and derived its Feynman rules starting from the Lagrangian mentioned above. We also discussed the additional effective couplings to matter fermions and gauge bosons occuring once bilinear RPVs are included.
In ch. 5 we rederived some known bounds on the gravitino mass from the observation of SN1987A. We presented the computation of the cross-section of the process $\gamma\gamma \longrightarrow \tilde{G}\tilde{G}$ in great detail, because we found several subtleties and complications concerning the usage of the equivalence theorem for light gravitinos, which had not been mentioned in the literature. We found similar bounds from photon

annihilation to the results by Riotto, Mohaparta and Grifols [1]. In particular we were able to exclude the gravitino mass range of

$$6.2 \times 10^{-8}\text{eV} < m_{3/2} < 1.8 \times 10^{-5}\text{eV}\,. \tag{7.1}$$

To obtain this result one has to assume very light scalar particles in the spectrum, the sgoldstinos.The corresponding diagrams are proportional to the large photino mass and dominate the total cross-section in the limit of superlight sgoldstinos. However, since the sgoldstino mass can also be very large in a variety of models, we performed the same calculation again with heavy sgoldstinos. In this scenario the resulting bounds are less restrictive because of the lower gravitino luminosity. Notably, other initial states can give rise to gravitino production rates similar to the ones via $\gamma\gamma \longrightarrow \tilde{G}\tilde{G}$. We showed this in the case of gravitino pair production via neutrino annihilation $\nu\overline{\nu} \to \tilde{G}\tilde{G}$. The obtained gravitino luminosity is of the same order as for photon annihilation with heavy sgoldstinos and we found the new result

$$6.0 \times 10^{-9}\text{eV} < m_{3/2} < 8.0 \times 10^{-7}\text{eV}\,. \tag{7.2}$$

In both limiting cases ($m_S, m_P \ll m_{\tilde{\gamma}}$ and $m_S, m_P \gg m_{\tilde{\gamma}}$) we find an excluded mass interval which covers two orders of magnitude and we can expect similar results for the intermediate case $m_S, m_P \sim m_{\tilde{\gamma}}$. The upper bound of $m_{3/2} < 8.0 \times 10^{-7}$eV is our most conservative result. It relies solely on interactions that are part of any SUGRA theory regardless of the specific model.
In the second part of our analysis we treated the production of single gravitinos in SN cores. For this to be possible R-parity cannot be conserved and we decided to include bilinear RPVs to our model. We found new gravitino production channels and calculated the cross-sections of the processes $\gamma\gamma \longrightarrow \tilde{G}\nu$ and $\nu\overline{\nu} \to \tilde{G}\nu$. The production rates turned out to be negligible compared to the ones due to the R-parity conserving channels, regardless of the sgoldstino masses.
We conclude that the observation of the SN neutrinos of SN1987A allows us to exclude a gravitino mass interval covering two orders of magnitude in the superlight mass regime. But the exact position of this interval depends on the specific model

and the various possible realizations of the hidden field sector and SUSY breaking. We also found that bilinear RPV would not affect these limits and only little can be learned about R-parity from SN observation in this way.
The results could be improved either by future SN observations or by new discoveries pointing towards specific SUGRA models with superlight gravitinos. A deeper understanding of SUSY breaking will allow us to set considerably more robust bounds on the gravitino mass based on SN observations.

A

Conventions, Notations, Natural Units and Physical Constants

Conventions and Notations

Spacetime Indices Greek letters $\mu, \nu ...$ denote spacetime indices of four-vectors and tensors. They can be raised or lowered using the metric tensor. For its signature we choose

$$(g^{\mu\nu}) = (g_{\mu\nu}) = \begin{pmatrix} +1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} . \tag{A.1}$$

For the partial derivative we use the notation

$$\partial_\mu \equiv \frac{\partial}{\partial x^\mu}\,, \quad \partial^\mu \equiv g^{\mu\nu}\partial_\nu\,. \tag{A.2}$$

(Anti-)Symmetrization of Lorentz indices are denoted with brackets,

$$T_{(\mu\nu)} \equiv \frac{1}{2}\left(T_{\mu\nu} + T_{\nu\mu}\right)\,, \tag{A.3}$$

$$T_{[\mu\nu]} \equiv \frac{1}{2}\left(T_{\mu\nu} - T_{\nu\mu}\right)\,. \tag{A.4}$$

In almost every case we use the Minkowski metric $\eta_{\mu\nu}$ of flat spacetime as our background.
Furthermore we fix the sign of the four-dimensional Levi-Civita symbol by

$$\epsilon_{0123} = -1\,. \tag{A.5}$$

Natural Units

Throughout this thesis we employ natural units. For this we set the speed of light, the reduced Planck constant and the Boltzmann constant to unity,

$$c = \hbar = k_B = 1\,. \tag{A.6}$$

In the case that we want to convert a final result back to SI-units we use the physical constants from this chapter and the following conversion relations,

Dimension	**Unit**	**Conversion Factor**	**Value**
Length	1eV^{-1}	$= 1\text{eV}^{-1}\hbar c$	$= 1.97 \times 10^{-7}\text{m}$
Mass	1eV	$= 1\text{eV}c^{-2}$	$= 1.78 \times 10^{-36}\text{kg}$
Time	1eV^{-1}	$= 1\text{eV}^{-1}\hbar$	$= 6.58 \times 10^{-16}\text{s}$
Temperature	1eV	$= 1\text{eV}k_B^{-1}$	$= 1.16 \times 10^{4}\text{ K}$

We will also use the non-SI energy unit **erg**,

$$1\text{erg} \equiv 10^{-7}\text{J} \approx 624\text{GeV}\,.$$

Physical Constants

We state the relevant physical constants for this thesis [36].

Constant	**Symbol**	**Value in SI Units**
speed of light in vacuum	c	299 792 458 m s^{-1}
reduced Planck constant	$\hbar = h/(2\pi)$	1.054 571 628(53) $\times 10^{-34}$ J s
electron charge	e	1.602 176 487(40) $\times 10^{-19}$ C
permittivity of free space	ϵ_0	8.854 187 817$\cdots \times 10^{-12}$ F m^{-1}
fine-structure constant	$\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c}$	1/137.035 999 679(94)
Newtonian gravitational constant	G_N	6.674 28(67) $\times 10^{-11}$ m^3 kg^{-1} s^{-2}
		$=$ 6.708 81(67) $\hbar\, c\,(\text{GeV}/c^2)^{-2}$
Fermi coupling constant	G_F	1.16637(1) $\times 10^{-5}$GeV^{-2}
Boltzmann constant	k_B	1.380 6504(24) $\times 10^{-23}$ J K^{-1}
weak-mixing angle	$\sin^2\theta_W(M_Z)$	0.231 16(13)
$W^{\pm}$ boson mass	m_W	80.399(23) GeV/c^2
Z^0 boson mass	m_Z	91.1876(21) GeV/c^2

Instead of the Newtonian constant G_N we use

$$\kappa \equiv \frac{1}{M_P}\,.$$

Here M_P is the reduced Planck mass given by

$$M_P \equiv \frac{m_P}{\sqrt{8\pi}}\,,\ \text{with the Planck mass}\ m_P \equiv \sqrt{\frac{\hbar c}{G_N}}\,.$$

In natural units we obtain

$$\kappa = \sqrt{8\pi G_N} = 4.11 \times 10^{-19}\ \text{GeV}^{-1}\,. \tag{A.7}$$

B

Spinors

B.1. Notations and Transformation Properties

Four-vectors and four-tensors can be defined by their transformation properties under the Lorentz group or more precisely their matrix representation of SO(3,1). The same goes for spinors.

We start our discussion of the spinor transformation properties with a complex 2×2 matrix Λ with $\det \Lambda = 1$, i.e. $\Lambda \in \mathrm{SL}(2, \mathbb{C})$ and a Hermitian 2×2 matrix P. The **Pauli matrices**

$$\sigma_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} , \quad \sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} , \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} , \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{B.1}$$

form a basis of the space of Hermitian 2×2 matrices. Therefore we can write[1]

$$P = \sum_{\mu=0}^{3} P_\mu \sigma^\mu = \begin{pmatrix} P_0 + P_3 & P_1 - iP_2 \\ P_1 + iP_2 & P_0 - P_3 \end{pmatrix} . \tag{B.2}$$

A transformation of P under $\mathrm{SL}(2,\mathbb{C})$ gives us a new Hermitian matrix P',

$$P \longmapsto P' = \Lambda P \Lambda^\dagger , \tag{B.3}$$

which can be expanded as in (B.2),

$$\begin{aligned} P'_\mu \sigma^\mu &= \Lambda P_\mu \sigma^\mu \Lambda^\dagger \\ \Rightarrow \det P'_\mu \sigma^\mu &= \det P_\mu \sigma^\mu \\ \Leftrightarrow P'^2_0 - P'^2_1 - P'^2_2 - P'^2_3 &= P_0^2 - P_1^2 - P_2^2 - P_3^2 . \end{aligned}$$

We see that P^μ and P'^μ are connected by a Lorentz transformation, the index μ is a proper Lorentz index. Any $\Lambda \in \mathrm{SL}(2,\mathbb{C})$ corresponds to a Lorentz transformation and $\mathrm{SL}(2,\mathbb{C})$ may be regarded as the group of Lorentz transformations for spinors.

Weyl Two-Spinors A left-handed Weyl-spinor ψ transforms in the $(\frac{1}{2},0)$ representation,

$$\psi_\alpha \longmapsto \psi'_\alpha = \Lambda_\alpha{}^\beta \psi_\beta ,$$

a right-handed Weyl spinor $\overline{\chi}$ in the conjugate $(0,\frac{1}{2})$ representation,

$$\overline{\chi}_{\dot\alpha} \longmapsto \psi'_{\dot\alpha} = \Lambda^{*}{}_{\dot\alpha}{}^{\dot\beta} \overline{\chi}_{\dot\beta} .$$

In order to differentiate between the two representations we employ the Van-der-Waerden notation [29].
Therefore the left and right-handed spinors are related by Hermitian conjugation,

$$(\psi_\alpha)^\dagger = \overline{\psi}_{\dot\alpha} , \quad (\overline{\chi}_{\dot\alpha})^\dagger = \chi_\alpha .$$

[1] The summation over μ is not a tensorial operation like $a_\mu b^\mu = \eta^{\mu\nu} a_\mu b_\nu$. The sum is written explicitly in order to avoid confusion with the summation convention.

The tensor $\epsilon_{\alpha\beta}$ with the components

$$\epsilon_{21} = \epsilon^{12} = 1\,, \quad \epsilon_{12} = \epsilon^{21} = -1\,, \quad \epsilon_{11} = \epsilon_{22} = 0$$

is invariant under $\mathrm{SL}(2,\mathbb{C})$,

$$\Lambda_\alpha{}^\beta \Lambda_\gamma{}^\delta \epsilon_{\beta\delta} = \epsilon_{\alpha\gamma}\,.$$

It is therefore called the spinor Minkowski metric and can be used to raise and lower spinor indices,

$$\psi^\alpha = \epsilon^{\alpha\beta}\psi_\beta\,, \quad \psi_\alpha = \epsilon_{\alpha\beta}\psi^\beta\,,$$

whose Lorentz transformations are given by

$$\begin{aligned}
\psi^\alpha &\longmapsto \psi'^\alpha = \left(\Lambda^{-1}\right)_\beta{}^\alpha \psi^\beta\,,\\
\overline{\chi}^{\dot\alpha} &\longmapsto \overline{\chi'}^{\dot\alpha} = \left(\Lambda^*\right)^{-1}{}_{\dot\beta}{}^{\dot\alpha}\overline{\chi}^{\dot\beta}\,.
\end{aligned}$$

The contraction of two anti-commuting Weyl spinors gives us a Lorentz scalar. By convention the notation is given by

$$\begin{aligned}
\psi\chi &\equiv \psi^\alpha\chi_\alpha = \epsilon^{\alpha\beta}\psi_\beta\chi_\alpha = -\epsilon^{\alpha\beta}\chi_\alpha\psi_\beta = \epsilon^{\beta\alpha}\chi_\alpha\psi_\beta = \chi^\beta\psi_\beta = \chi\psi\,,\\
\overline{\psi}\overline{\chi} &\equiv \overline{\psi}_{\dot\alpha}\overline{\chi}^{\dot\alpha} = \epsilon^{\dot\alpha\dot\beta}\overline{\psi}_{\dot\alpha}\overline{\chi}_{\dot\beta} = -\epsilon^{\dot\alpha\dot\beta}\overline{\chi}_{\dot\beta}\overline{\psi}_{\dot\alpha} = \epsilon^{\dot\beta\dot\alpha}\overline{\chi}_{\dot\beta}\overline{\psi}_{\dot\alpha} = \overline{\chi}_{\dot\beta}\overline{\psi}^{\dot\beta} = \overline{\chi}\overline{\psi}\,.
\end{aligned}$$

We can also raise and lower the spinor indices of the Pauli matrices and therefore define

$$\begin{aligned}
&\overline{\sigma}^{\mu\dot\alpha\alpha} \equiv \epsilon^{\dot\alpha\dot\beta}\epsilon^{\alpha\beta}\sigma^\mu{}_{\beta\dot\beta}\\
&\Rightarrow \sigma^\mu = \begin{pmatrix} \mathbb{1} \\ \vec\sigma \end{pmatrix}\,, \quad \overline{\sigma}^\mu = \begin{pmatrix} \mathbb{1} \\ -\vec\sigma \end{pmatrix}\,.
\end{aligned}$$

Dirac Bi-Spinors Combining a left-handed with a right-handed spinor $\left(\frac{1}{2},0\right)\oplus\left(0,\frac{1}{2}\right)$ we obtain a four-component Dirac bi-spinor,

$$\Psi_D = \begin{pmatrix} \psi_\alpha \\ \overline{\chi}^{\dot{\alpha}} \end{pmatrix} .$$

We adopt the conventions of [73]. The γ-matrices

$$\gamma_\mu = \begin{pmatrix} 0 & \sigma_\mu \\ \overline{\sigma}_\mu & 0 \end{pmatrix} \tag{B.4}$$

satisfy the Clifford algebra

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu} . \tag{B.5}$$

The Feynman slash notation,

$$\not{p} \equiv \gamma^\mu p_\mu \tag{B.6}$$

is used throughout the thesis. The chiral projectors are given by

$$P_L = \frac{1}{2}\left(\mathbb{1} - \gamma^5\right) , \quad \text{and} \quad P_R = \frac{1}{2}\left(\mathbb{1} + \gamma^5\right) , \tag{B.7}$$

where

$$\gamma^5 = -\frac{i}{4!}\epsilon^{\mu\nu\rho\sigma}\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} -\mathbb{1} & 0 \\ 0 & \mathbb{1} \end{pmatrix} . \tag{B.8}$$

Under Lorentz transformations a Dirac bi-spinor transforms like

$$\Psi(x) \mapsto \Psi'(x) = \Lambda_{1/2}\Psi(x) \equiv \exp\left[-\frac{i}{2}\Theta^{\mu\nu}S_{\mu\nu}\right]\Psi(x) . \tag{B.9}$$

The Lorentz generators are given by

$$S^{\mu\nu} = \frac{1}{2}\sigma^{\mu\nu} , \tag{B.10}$$

where

$$\sigma^{\mu\nu} = \frac{i}{2}\left[\gamma^\mu, \gamma^\nu\right] . \tag{B.11}$$

We define the charge conjugation matrix C,

$$C = i\gamma^2\gamma^0 = \begin{pmatrix} i\sigma_2 & 0 \\ 0 & -i\sigma_2 \end{pmatrix} \tag{B.12}$$

as well as the adjoint and the charge conjugate of Ψ_D,

$$\overline{\Psi}_D \equiv \Psi_D^\dagger \gamma^0 = \begin{pmatrix} \chi^\alpha & \overline{\psi}_{\dot{\alpha}} \end{pmatrix} .$$

$$\Psi_D^c \equiv C\overline{\Psi}_D^T = \begin{pmatrix} \chi_\alpha \\ \overline{\psi}^{\dot{\alpha}} \end{pmatrix} .$$

The charge conjugation satisfies

$$C^\dagger = C^T = C^{-1} = -C\,, \quad C^{-1}\gamma_\mu C = -\gamma_\mu^T\,, \quad C^{-1}\gamma^5 C = \gamma^{5T} . \tag{B.13}$$

Its action on spinors reads

$$u(p) = C\overline{v}^T(p)\,, \quad v(p) = C\overline{u}^T(p) \tag{B.14}$$

for Dirac and Majorana spinors. A **Majorana spinor** [71] satisfies the reality condition $\Psi_M = \Psi_M^c$, hence we write it as

$$\Psi_M = \begin{pmatrix} \psi_\alpha \\ \overline{\psi}^{\dot{\alpha}} \end{pmatrix} , \tag{B.15}$$

$$\overline{\Psi}_M = \begin{pmatrix} \psi^\alpha & \overline{\psi}_{\dot{\alpha}} \end{pmatrix} . \tag{B.16}$$

This can also be written as

$$v(p) = \gamma_0 C u^*(p)\,, \quad u(p) = \gamma_0 C v^*(p)\,. \tag{B.17}$$

B.2. Spinors in Curved Spacetimes

In the case of General Relativity we deal with curved spacetime. For the treatment of spinors in curved spacetime we have to get a little more involved and introduce the formulation of GR in terms of the **vielbein** [74]. This formalism is necessary for the derivation of graviton fermion vertices, see app. C.

Vielbein, Spin Connection and Covariant Derivative of Spinors

At any point of spacetime, we are able to choose a frame of reference, such that the local metric is that of flat spacetime η_{ab}[2]. The two systems are connected by the vielbein ${e_\mu}^a$ and its inverse ${e^\mu}_a$,

$$ {e^\mu}_a {e_\nu}^a = \delta^\mu_\nu \,, \quad {e_\mu}^a {e^\mu}_b = \delta^a_b \,. \tag{B.18} $$

The vielbein connects the metric of the globally curved spacetime with the local Minkowski metric,

$$ \eta_{ab} = g_{\mu\nu} {e^\mu}_a {e^\nu}_b \,, \quad g_{\mu\nu} = {e_\mu}^a {e_\nu}^b \eta_{ab} \,. \tag{B.19} $$

For this reason the vielbein is sometimes referred to as the 'square root' of the metric. We raise and lower flat indices with η_{ab} and curved ones with $g_{\mu\nu}$.
The covariant derivative of a tensor in the coordinate basis is given by the Christoffel symbols,

$$ D_\mu V^\alpha = \partial_\mu V^\alpha + \Gamma^\alpha_{\nu\mu} V^\nu \,. \tag{B.20} $$

In our flat frame we have to use the **spin connection** ${\omega_\mu}^a{}_b$ instead of the Christoffel symbols[3],

$$ D_\mu V^a = \partial_\mu V^a - {\omega_\mu}^a{}_c V^c \,. \tag{B.21} $$

[2]In this section we will always distinct between Einstein indices $\mu, \nu ...$ and flat indices $a, b, ...$
[3]The overall sign of the spin connection can differ from reference to reference.

We will need the spin connection in particular to write a covariant derivative of spinors, hence the name. By comparing (B.20) to (B.21) one can show that the spin connection is related to the Christoffel symbols by

$$\Gamma^{\nu}_{\mu\lambda} = e^{\nu}{}_{a}\partial_{\mu}e_{\lambda}{}^{a} - e^{\nu}{}_{a}e_{\lambda}{}^{b}\omega_{\mu}{}^{a}{}_{b}\,, \tag{B.22}$$

$$\Leftrightarrow \omega_{\mu}{}^{a}{}_{b} = e^{\lambda}{}_{b}\left(\partial_{\mu}e_{\lambda}{}^{a} - e_{\nu}{}^{a}\Gamma^{\nu}_{\mu\lambda}\right)\,. \tag{B.23}$$

This can be re-expressed as the so-called 'tetrad postulate',

$$D_{\mu}e_{\nu}{}^{a} \equiv \partial_{\mu}e_{\nu}{}^{a} - e_{\sigma}{}^{a}\Gamma^{\sigma}_{\mu\nu} - \omega_{\mu}{}^{a}{}_{b}e_{\nu}{}^{b} = 0\,, \tag{B.24}$$

which leads directly to metric compatibility, $D_{\mu}g_{\nu\lambda} = 0$.
We can express the spin-connection as a function of the vielbein only,

$$\begin{aligned}\omega_{\mu ab} = {} & \frac{1}{2}e^{\nu}{}_{a}\left(\partial_{\nu}e_{\mu b} - \partial_{\mu}e_{\nu b}\right) + \frac{1}{2}e^{\nu}{}_{b}\left(\partial_{\mu}e_{\nu a} - \partial_{\nu}e_{\mu a}\right) \\ & + \frac{1}{2}e^{\rho}{}_{a}e^{\sigma}{}_{b}\left(\partial_{\rho}e_{\sigma c} - \partial_{\sigma}e_{\rho c}\right)e^{c}{}_{\mu}\,.\end{aligned} \tag{B.25}$$

Covariant Derivative of a Fermion Field In GR the Lorentz transformation parameter $\theta^{\mu\nu}$ in (B.9) become spacetime dependent. As usual the partial derivative now transforms like

$$\partial_{\mu}\Psi(x) \mapsto \partial_{\mu}\Psi'(x) \neq \Lambda_{1/2}\partial_{\mu}\Psi(x)\,, \tag{B.26}$$

and we need a covariant derivative including a connection [75]

$$D_{\mu}\Psi(x) = \left(\partial_{\mu} + \Omega_{\mu}\right)\Psi(x)\,, \tag{B.27}$$

$$\text{with } \Omega_{\mu} \mapsto \Lambda_{1/2}(x)\Omega_{\mu}\Lambda^{-1}_{1/2}(x) - \left(\partial_{\mu}\Lambda_{1/2}(x)\right)\Lambda^{-1}_{1/2}(x)\,, \tag{B.28}$$

such that

$$D_{\mu}\Psi(x) \mapsto \Lambda_{1/2}(x)D_{\mu}\Psi(x)\,. \tag{B.29}$$

The connection is given by

$$\Omega_\mu = \frac{i}{2}\omega_\mu{}^{ab} S_{ab} \,. \tag{B.30}$$

The covariant derivative of a spinor field is correspondingly

$$D_\mu \Psi(x) = \left(\partial_\mu + \frac{i}{4}\omega_\mu{}^{ab}\sigma_{ab}\right)\Psi(x) \,. \tag{B.31}$$

C
Linearized Gravity and Graviton Feynman Rules

C.1. The Weak-Field-Approximation of Gravity

In situations, where gravitational interactions are weak, we may neglect non-linear contributions from the Einstein-Hilbert action. This is called **linearized gravity**. Starting from the Lagrangian of some field theory we insert the spacetime dependent metric $g_{\mu\nu}(x)$ for any Minkowski metric $\eta_{\mu\nu}$, promote partial derivatives to covariant derivatives and replace d^4x by the covariant volume element $\sqrt{-g}\mathrm{d}^4x$. We choose some solution of the Einstein equations as our classical background, for this we always choose the Minkowski metric $\eta_{\mu\nu}$ of flat spacetime, and consider small fluctuations or quantum contributions [76]. This is called the weak-field-approximation

$$g_{\mu\nu} = \eta_{\mu\nu} + 2\kappa h_{\mu\nu} \Longrightarrow g^{\mu\nu} = \eta^{\mu\nu} - 2\kappa h^{\mu\nu} + \mathcal{O}(\kappa^2)\,, \tag{C.1}$$

where $\kappa = \sqrt{8\pi G_N}$ and $h_{\mu\nu}$ is the graviton field [1]. We assume that this field is symmetric in its indices. It also appears hidden in the metric's and vielbein's determinant,

$$\sqrt{-g} = e = 1 + \kappa h + \mathcal{O}(\kappa^2)\,, \quad \text{where} \quad h \equiv \eta_{\mu\nu} h^{\mu\nu}\,. \tag{C.2}$$

For fermionic fields it is necessary to perform the weak-field-approximation in terms of the vielbein as well, we write

$$e_\mu{}^a = \delta_\mu^a + \kappa c_\mu{}^a + \mathcal{O}\left(\kappa^2\right)\,, \tag{C.3}$$

$$e^\mu{}_a = \delta_a^\mu - \kappa c^\mu{}_a + \mathcal{O}\left(\kappa^2\right)\,. \tag{C.4}$$

Combining the equations (B.19) and (C.1) we find

$$g_{\mu\nu} = e_\mu{}^a e_\nu{}^b \eta_{ab} = \eta_{\mu\nu} + \kappa(c_{\mu\nu} + c_{\nu\mu}) + \mathcal{O}\left(\kappa^2\right) \stackrel{!}{=} \eta_{\mu\nu} + 2\kappa h_{\mu\nu} + \mathcal{O}(\kappa^2)\,. \tag{C.5}$$

Therefore the graviton field is given by $h_{\mu\nu} = \frac{1}{2}\left(c_{\mu\nu} + c_{\nu\mu}\right) = c_{(\mu\nu)}$. We are only interested in the symmetric part of $c_{\mu\nu}$ and can always perform the substitution [76]

$$c_{\mu\nu} \mapsto c_{(\mu\nu)} = h_{\mu\nu}\,. \tag{C.6}$$

Alternatively and equivalently we could just impose the symmetry $c_{\mu\nu} = c_{\nu\mu}$, which is called the Lorentz symmetric gauge [78].
At last we can expand the spin-connection (B.25) as

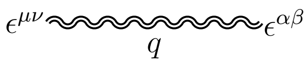
$$\omega_{\mu ab} = \kappa\left(\partial_a h_{\mu_b} - \partial_b h_{\mu a}\right) + \mathcal{O}(\kappa^2)\,. \tag{C.7}$$

The field $h_{\mu\nu}$ describes the spin-2 graviton field. In this chapter we want to derive the relevant Feynman rules of the graviton's interactions with matter.

[1] Please note that some authors define the gravitational coupling κ as $\sqrt{32\pi G_N}$ [76] or $\sqrt{16\pi G_N}$ [77] instead of (A.7). Also some authors define the graviton field with an additional factor of $\sqrt{2}$, compared to our $h_{\mu\nu}$ [77].

C.2. Graviton Propagator

We will only state the graviton's propagator in the harmonic (or De Donder) gauge without the derivation, which can be found e.g. in [77].



$$= \frac{i}{2}\frac{\left(\eta_{\mu\alpha}\eta_{\nu\beta} + \eta_{\mu\beta}\eta_{\nu\alpha} - \eta_{\mu\nu}\eta_{\alpha\beta}\right)}{q^2}\,, \tag{C.8}$$

Compared to Veltman's famous lecture notes [77] we have an additional factor of $\frac{1}{2}$ caused by the different definitions of the graviton field,

$$h_{\mu\nu} = \frac{1}{\sqrt{2}} h_{\mu\nu}^{\text{Veltman}}\,. \tag{C.9}$$

C.3. Graviton Interactions with Matter

In order to derive the vertices of the graviton interactions to matter we substitute the global curved spacetime metric $g_{\mu\nu}$ and its determinant appearing in a Lagrangian with (C.1) and (C.2).
As an alternative derivation or a consistency check we can also write the interaction part as

$$-\kappa h_{\mu\nu} T^{\mu\nu} + \mathcal{O}\left(\kappa^2\right)\,, \tag{C.10}$$

where $T^{\mu\nu}$ is the **symmetric** Hilbert energy momentum tensor (EMT)

$$T^{\mu\nu} = \frac{2}{\sqrt{-g}}\frac{\delta \mathcal{L}\sqrt{-g}}{\delta g_{\mu\nu}}\,. \tag{C.11}$$

For a particles without spin this tensor corresponds to the **canonical** EMT coming from the Noether theorem,

$$\Theta^{\mu\nu} = \frac{\partial \mathcal{L}_{\text{matter}}}{\partial(\partial_\nu \Phi_i)}\partial^\mu \Phi_i - g^{\mu\nu}\mathcal{L}_{\text{matter}}\,,$$

where i runs over the matter fields in the Lagrangian $\mathcal{L}_{\text{matter}}$. In general the canonical EMT does not need need to be symmetric nor gauge invariant. It can however be 'fixed' by a method of Belinfante and Rosenfeld [79] which is described e.g. in [73, 80]. For this terms are added to $\Theta^{\mu\nu}$ which do not spoil the conservation law and leave us with a symmetric and gauge invariant tensor,

$$T^{\mu\nu} = \Theta^{\mu\nu} + \partial_\lambda K^{\mu\nu\lambda}\,, \quad \text{with } K^{\mu\nu\lambda} = -K^{\mu\lambda\nu}\,. \tag{C.12}$$

But it should be stated here that the correct symmetric EMT can also be derived from Noether's theorem alone [81].

We will now work out four examples, which clarify the derivation of graviton vertices, starting with the simplest. Note that in all of the following vertices the momenta are assumed to flow into the vertex as usual.

Real Scalar Field

The Lagrangian of a massive real scalar field ϕ in curved spacetime is given by

$$\mathcal{L} = e\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - e\frac{m_s^2}{2}\phi^2\,. \tag{C.13}$$

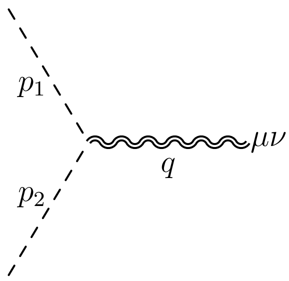
For scalar fields we find that $T^{\mu\nu} = \Theta^{\mu\nu}$, since

$$\Theta^{\mu\nu} = e\partial^\mu\phi\partial^\nu\phi + \frac{1}{2}eg^{\mu\nu}\left(m^2\phi^2 - \partial^\lambda\phi\partial_\lambda\phi\right) \tag{C.14}$$

is symmetric already.
Now we insert (C.1) and (C.2) into the Lagrangian and find

$$\begin{aligned}\mathcal{L} &= \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{m^2}{2}\phi^2 + \frac{\kappa}{2}h\left(\partial_\mu\phi\partial^\mu\phi - m_s^2\phi^2\right) && \text{(C.15)}\\ &\quad - \kappa h^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \mathcal{O}\left(\kappa^2\right) \\ &= \mathcal{L}^{(0)} - \kappa h^{\mu\nu}\left[\partial_\mu\phi\partial_\nu\phi + \frac{1}{2}\eta_{\mu\nu}\left(m_s^2\phi^2 - \partial^\lambda\phi\partial_\lambda\phi\right)\right] + \mathcal{O}\left(\kappa^2\right) && \text{(C.16)}\\ &\equiv \mathcal{L}^{(0)} - \kappa h^{\mu\nu}T_{\mu\nu} + \mathcal{O}\left(\kappa^2\right)\,. && \text{(C.17)}\end{aligned}$$

Here we defined $\mathcal{L}^{(0)} \equiv \mathcal{L}|_{\kappa=0}$. The term in the brackets in (C.16) is exactly the canonical EMT (C.14). The vertex rule is



$$= i\kappa \left[p_{1\mu}p_{2\nu} + p_{1\nu}p_{2\mu} - \eta_{\mu\nu}\left(p_1 \cdot p_2 + m_s^2\right)\right] . \qquad \text{(C.18)}$$

Vector Field

We start from the Lagrangian of the electromagnetic field,

$$\sqrt{-g}\mathcal{L} = -\frac{1}{4}\sqrt{-g}F_{\mu\nu}F^{\mu\nu} = -\frac{1}{4}\sqrt{-g}g^{\mu\alpha}g^{\nu\beta}F_{\mu\nu}F_{\alpha\beta} . \qquad \text{(C.19)}$$

The canonical EMT reads

$$\begin{aligned} \Theta^{\mu\nu} =& e\frac{\partial\mathcal{L}}{\partial(\partial_\nu A_\lambda)}\partial^\mu A_\lambda - eg^{\mu\nu}\mathcal{L} \\ =& eF^{\lambda\nu}\partial^\mu A_\lambda + e\frac{1}{4}g^{\mu\nu}F_{\alpha\beta}F^{\alpha\beta} , \end{aligned} \qquad \text{(C.20)}$$

which is obviously not symmetric in its two indices. We use Belinfante's method to fix this by adding the term $-F^{\lambda\nu}\partial_\lambda A^\mu$ and find the symmetric EMT of the electromagnetic field in flat spacetime,

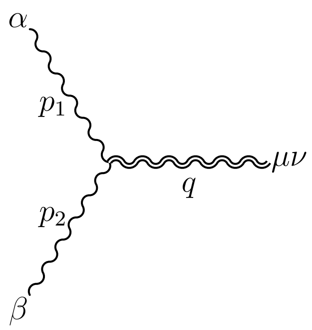
$$T_{\mu\nu} = \eta^{\alpha\beta}F_{\mu\alpha}F_{\beta\nu} + \frac{1}{4}\eta_{\mu\nu}F_{\alpha\beta}F^{\alpha\beta} \qquad \text{(C.21)}$$

$$\begin{aligned} &= \eta_{\mu\alpha}\partial_\beta A^\alpha\partial_\nu A^\beta + \eta_{\nu\beta}\partial_\mu A^\alpha\partial_\alpha A^\beta - \eta_{\alpha\beta}\partial_\mu A^\alpha\partial_\nu A^\beta - \eta_{\mu\alpha}\eta_{\nu\beta}\partial^\lambda A^\alpha\partial_\lambda A^\beta \\ &\quad + \frac{1}{2}\eta_{\mu\nu}\left(\eta_{\alpha\beta}\partial^\lambda A^\alpha\partial_\lambda A^\beta - \partial_\beta A^\alpha\partial_\alpha A^\beta\right) . \end{aligned} \qquad \text{(C.22)}$$

For the interaction vertex with the graviton we expand (C.19) using (C.1) and (C.2).

$$
\begin{aligned}
\sqrt{-g}\mathcal{L} &= -\frac{1}{4}\sqrt{-g}g^{\mu\alpha}g^{\nu\beta}F_{\mu\nu}F_{\alpha\beta} \\
&= \mathcal{L}^{(0)} - \frac{\kappa}{4}hF_{\alpha\beta}F^{\alpha\beta} - \frac{\kappa}{2}\left(\eta^{\nu\beta}h^{\mu\alpha} - \eta^{\mu\alpha}h^{\nu\beta}\right)F_{\alpha\beta}F_{\mu\nu} + \mathcal{O}\left(\kappa^2\right) \\
&= \mathcal{L}^{(0)} - \kappa h^{\mu\nu}\left[\eta^{\alpha\beta}F_{\mu\alpha}F_{\beta\nu} + \frac{1}{4}\eta_{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}\right] + \mathcal{O}\left(\kappa^2\right) \\
&\equiv \mathcal{L}^{(0)} - \kappa h^{\mu\nu}T_{\mu\nu} + \mathcal{O}\left(\kappa^2\right)\ .
\end{aligned}
\tag{C.23}
$$

Again we recognize the term in the brackets as the EMT (C.21). By looking at (C.22) we can read of the vertex rule.



$$
= \quad i\kappa\Big[\eta_{\mu\alpha}p_{1\beta}p_{2\nu} + \eta_{\nu\alpha}p_{1\beta}p_{2\mu} + \eta_{\nu\beta}p_{1\mu}p_{2\alpha} + \eta_{\mu\beta}p_{1\nu}p_{2\alpha} - \eta_{\alpha\beta}\left(p_{1\mu}p_{2\nu} + p_{1\nu}p_{2\mu}\right) - (p_1\cdot p_2)(\eta_{\mu\alpha}\eta_{\nu\beta} + \eta_{\nu\alpha}\eta_{\mu\beta}) + \eta_{\mu\nu}\left((p_1\cdot p_2)\eta_{\alpha\beta} - p_{1\beta}p_{2\alpha}\right)\Big]\ . \tag{C.24}
$$

This is in agreement with [55] considering their deviating definition of the graviton field, see (C.9).

Dirac Field

For graviton vertices with **fermionic fields** we have to employ the formulation of GR in terms of the **vielbein** introduced in sec. B.2 and its weak field approximation, see sec. C.1 and [82]. For the derivation of the fermion-graviton coupling the correct Lagrangian of a massive spin $\frac{1}{2}$ field takes the form [83]

$$
\sqrt{-g}\mathcal{L}_{\text{Dirac}} = \sqrt{-g}\left(\frac{i}{2}\overline{\psi}e^{\mu}{}_{a}\gamma^{a}\overleftrightarrow{D}_{\mu}\psi - m_f\overline{\psi}\psi\right)\ . \tag{C.25}
$$

Here we define $\overline{\psi}\overleftrightarrow{D}_\mu\psi = \overline{\psi}D_\mu\psi - \overline{D_\mu\psi}\psi$ and express it as,

$$\begin{aligned}\overline{\psi}\gamma_\nu\overleftrightarrow{D}_\mu\psi &= \overline{\psi}\gamma_\nu\left(\partial_\mu + \frac{i}{4}\omega_\mu{}^{ab}\sigma_{ab}\right) - \left[\left(\partial_\mu + \frac{i}{4}\omega_\mu{}^{ab}\sigma_{ab}\right)\psi\right]^\dagger\gamma^0\gamma_\nu\psi \\ &= \overline{\psi}\gamma_\nu\overleftrightarrow{\partial}_\mu\psi + \frac{i}{4}\omega_\mu{}^{ab}\overline{\psi}\left\{\gamma_\nu,\sigma_{ab}\right\}\psi\,. \end{aligned} \tag{C.26}$$

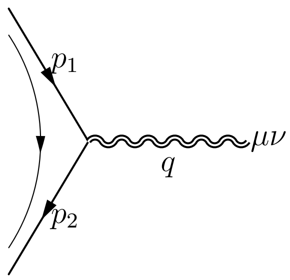
Next we substitute (B.31),(C.2), (C.4) and (C.7) to obtain the Lagrangian,

$$\begin{aligned}\sqrt{-g}\mathcal{L}_{\text{Dirac}} = &\left(\frac{i}{2}\overline{\psi}\gamma^\lambda\overleftrightarrow{\partial}_\lambda\psi - m_f\overline{\psi}\psi\right) - \kappa h^{\mu\nu}\left[\frac{i}{2}\overline{\psi}\gamma_{(\nu}\overleftrightarrow{\partial}_{\mu)}\psi - \eta_{\mu\nu}\left(\frac{i}{2}\overline{\psi}\gamma^\lambda\overleftrightarrow{\partial}_\lambda\psi - m_f\overline{\psi}\psi\right)\right] \\ &+ \frac{\kappa}{4}\partial^\lambda h^{\mu\nu}\overline{\psi}\left\{\gamma_\mu,\sigma_{\nu\lambda}\right\}\psi + \mathcal{O}(\kappa^2)\,.\end{aligned} \tag{C.27}$$

The last term vanishes due to the fact that the graviton field is symmetric and

$$\left\{\gamma_\mu,\sigma_{\nu\lambda}\right\} = -\left\{\gamma_\nu,\sigma_{\mu\lambda}\right\}\,, \tag{C.28}$$

which can be shown by a short calculation using (B.5). From (C.27) we read off the vertex,



$$= \frac{i\kappa}{2}\left(\gamma_{(\mu}(p_1-p_2)_{\nu)} + \eta_{\mu\nu}\left[(\not{p}_2 - \not{p}_1) - 2m_f\right]\right)\,.$$

Rarita-Schwinger Field

The Lagrangian of the spin-$\frac{3}{2}$ field in curved spacetime is given by

$$\sqrt{-g}\mathcal{L} = -\frac{1}{2}\epsilon^{\mu\nu\kappa\lambda}\overline{\psi}_\mu\gamma^5\gamma_\nu\overleftrightarrow{D}_\kappa\psi_\lambda + \sqrt{-g}\frac{i}{2}m_{3/2}\overline{\psi}_\mu\sigma^{\mu\nu}\psi_\nu\,, \tag{C.29}$$

where the covariant derivative is given by the spin connection, $D_\mu\psi_\lambda = \left(\partial_\mu + \frac{i}{4}\omega_\mu{}^{ab}\sigma_{ab}\right)\psi_\lambda$, see (B.31).

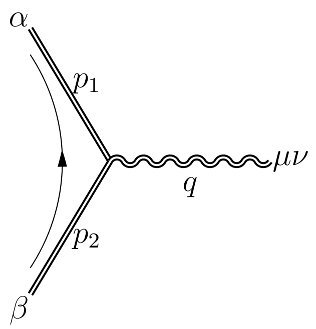
Since we deal with on-shell gravitinos the field equation (4.4) will help us to simplify our vertex factor,

$$\overline{\psi}_\mu \sigma^{\mu\nu} \psi_\nu = -ig^{\mu\nu} \overline{\psi}_\mu \psi_\nu \,. \tag{C.30}$$

With the insertion of the covariant derivative and graviton field, we obtain

$$\begin{aligned} \sqrt{-g}\mathcal{L} = \mathcal{L}^{(0)} &- \frac{\kappa}{2}\epsilon_{\alpha\rho\beta\nu} h^{\mu\nu} \overline{\psi}^\alpha \gamma^5 \gamma_\mu \overleftrightarrow{\partial}^\rho \psi^\beta - \frac{i}{4}\partial^\rho h^{\mu\nu} \epsilon_{\alpha\lambda\beta\mu} \overline{\psi}^\alpha \gamma^5 \left\{\gamma^\lambda, \sigma_{\nu\rho}\right\} \psi^\beta \\ &+ \frac{\kappa m_{3/2}}{2} h^{\mu\nu} \left(\eta_{\mu\nu}\eta_{\alpha\beta} \overline{\psi}^\alpha \psi^\beta - 2\eta_{\alpha\mu}\eta_{\nu\beta} \overline{\psi}^\alpha \psi^\beta\right) + \mathcal{O}(\kappa^2) \,. \end{aligned} \tag{C.31}$$

Here we not only performed the steps shown in (C.26) but also used the relation $[\gamma^5, \sigma^{\mu\nu}] = 0$. Completing this appendix we read off the vertex factor,



$$= \frac{\kappa}{2}\left[\epsilon_{\alpha\rho\beta(\mu}\gamma^5\gamma_{\nu)}(p_2 - p_1)^\rho + \frac{i}{2}\epsilon_{\alpha\lambda\beta(\mu}\gamma^5 \left\{\gamma^\lambda, \sigma_{\nu)\rho}\right\} q^\rho \right. \\ \left. + 2im_{3/2}\left(\eta_{\mu\nu}\eta_{\alpha\beta} - 2\eta_{\alpha(\mu}\eta_{\nu)\beta}\right)\right] .$$

This vertex coincides with the one given in [55], it is however important to bear in mind that these authors employ the Veltman definition of the graviton field, see (C.9).

D

Kinematics and Cross-Sections of Two-Body-Scatterings

Kinematics

We describe the general kinematics of $2 \longrightarrow 2$ scatterings in the center-of-mass frame of reference defined by $\sum_i \vec{p}_i = \vec{0}$. The setting is depicted in figure D.1.We start by writing the four momenta as

$$p_1 = \begin{pmatrix} E_1 \\ \vec{p} \end{pmatrix} , \quad p_2 = \begin{pmatrix} E_2 \\ -\vec{p} \end{pmatrix} , \tag{D.1}$$

$$k_1 = \begin{pmatrix} E_3 \\ \vec{k} \end{pmatrix} , \quad k_2 = \begin{pmatrix} E_4 \\ -\vec{k} \end{pmatrix} . \tag{D.2}$$

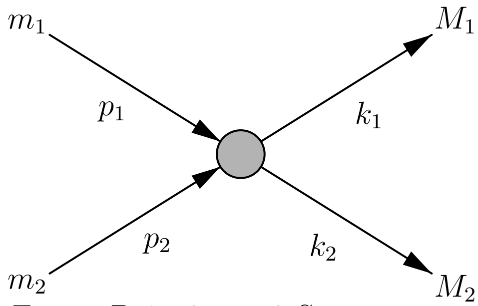


Figure D.1.: $2 \longrightarrow 2$ Scattering

The norm of the spatial momenta are given by

$$p \equiv |\vec{p}| = \frac{1}{2}\sqrt{\frac{m_1^4 + (s - m_2^2)^2 - 2m_1^2(s + m_2^2)}{s}}\,, \tag{D.3}$$

$$k \equiv \left|\vec{k}\right| = \frac{1}{2}\sqrt{\frac{M_1^4 + (s - M_2^2)^2 - 2M_1^2(s + M_2^2)}{s}}\,, \tag{D.4}$$

and the energies by

$$E_1 = \frac{1}{2}\sqrt{\frac{(m_1^2 - m_2^2 + s)^2}{s}}\,, \quad E_2 = \frac{1}{2}\sqrt{\frac{(m_2^2 - m_1^2 + s)^2}{s}}\,, \tag{D.5}$$

$$E_3 = \frac{1}{2}\sqrt{\frac{(M_1^2 - M_2^2 + s)^2}{s}}\,, \quad E_4 = \frac{1}{2}\sqrt{\frac{(M_2^2 - M_1^2 + s)^2}{s}}\,. \tag{D.6}$$

In calculations of scattering cross-sections we will need the scalar products of the momenta,

$$p_1 \cdot p_1 = m_1^2\,, \quad p_2 \cdot p_2 = m_2^2\,, \quad k_1 \cdot k_1 = M_1^2\,, \quad k_2 \cdot k_2 = M_2^2\,, \tag{D.7}$$

$$p_1 \cdot p_2 = E_1E_2 + p^2\,, \qquad p_1 \cdot k_1 = E_1E_3 - pk\cos\theta\,, \tag{D.8}$$

$$p_1 \cdot k_2 = E_1E_4 + pk\cos\theta\,, \quad p_2 \cdot k_1 = E_2E_3 + pk\cos\theta\,, \tag{D.9}$$

$$p_2 \cdot k_2 = E_2E_4 - pk\cos\theta\,, \quad k_1 \cdot k_2 = E_3E_4 + k^2\,, \tag{D.10}$$

where $\theta = \measuredangle(\vec{p}, \vec{k})$.

Kinematics of Gravitino Pair Production With the massless photons or almost massless neutrinos, $m_1 = m_2 = 0$ and the gravitinos $M_1 = M_2 = m_{3/2}$, we obtain

$$p_1 \cdot p_1 = p_2 \cdot p_2 = 0\,, \quad k_1 \cdot k_1 = k_2 \cdot k_2 = m_{3/2}^2\,, \tag{D.11}$$

$$p_1 \cdot p_2 = \frac{s}{2}\,, \quad k_1 \cdot k_2 = \frac{s}{2} - m_{3/2}^2\,, \tag{D.12}$$

$$p_1 \cdot k_1 = p_2 \cdot k_2 = \frac{s}{4} - \frac{\sqrt{s}}{2}k\cos\theta\,, \tag{D.13}$$

$$p_1 \cdot k_2 = p_2 \cdot k_1 = \frac{s}{4} + \frac{\sqrt{s}}{2}k\cos\theta\,, \tag{D.14}$$

with $k = \sqrt{\frac{s}{4} - m_{3/2}^2}$. In using the equivalence theorem later on, we will treat the massive gravitino as a massless goldstino. With $k = \frac{\sqrt{s}}{2}$ the following, more simple relations hold,

$$p_1 \cdot p_1 = p_2 \cdot p_2 = k_1 \cdot k_1 = k_2 \cdot k_2 = 0\,, \quad p_1 \cdot p_2 = \frac{s}{2}\,, \quad k_1 \cdot k_2 = \frac{s}{2}\,, \tag{D.15}$$

$$p_1 \cdot k_1 = p_2 \cdot k_2 = \frac{s}{4}\,(1 - \cos\theta)\ , \quad p_1 \cdot k_2 = p_2 \cdot k_1 = \frac{s}{4}\,(1 + \cos\theta)\ . \tag{D.16}$$

Unpolarized Scattering Cross-Section

We start with the expression for the cross-section of a general scattering process $p_1 p_2 \longrightarrow k_1 k_2$ [73],

$$\begin{aligned} \mathrm{d}\sigma = & \frac{1}{2p_1^0 2p_2^0 |v_1 - v_2|} \frac{\mathrm{d}^3 k_1}{(2\pi)^3\, 2k_1^0} \frac{\mathrm{d}^3 k_2}{(2\pi)^3\, 2k_2^0} \\ & \times (2\pi)^4 \delta^{(4)}(p_1 + p_2 - k_1 - k_2) \overline{|\mathcal{M}(p_1, p_2 \to k_1, k_2)|^2}\,, \end{aligned} \tag{D.17}$$

The line over the squared amplitude indicates, that we already summed over the final and took the average of the initial spins. The relative velocity appearing in the denominator is given [84] by

$$|v_1 - v_2| = \frac{1}{E_1 E_2} \sqrt{(p_1 \cdot p_2)^2 - m_1^2 m_2^2}\,, \tag{D.18}$$

which becomes

$$|v_1 - v_2| = \frac{|p|(E_1 + E_2)}{E_1 E_2} = \frac{|p|}{E_2} + \frac{|p|}{E_1} \tag{D.19}$$

in the center-of-mass system.
When calculating total cross-sections by integration of (D.17), we avoid counting identical final states several times by dividing by $N_{\mathrm{id}}!$, where N_{id} is the number of identical final state particles, otherwise we would count physically indistinguishable

events more than once.

$$\sigma_{\text{total}} = \frac{1}{N_{\text{id}}!} \frac{1}{2p_1^0 2p_2^0 |v_1 - v_2|} \int \frac{\mathrm{d}^3 k_1}{(2\pi)^3\, 2k_1^0} \int \frac{\mathrm{d}^3 k_2}{(2\pi)^3\, 2k_2^0}$$
$$(2\pi)^4 \delta^{(4)}(p_1 + p_2 - k_1 - k_2) \overline{|\mathcal{M}(p_1, p_2 \to k_1, k_2)|^2}\,. \tag{D.20}$$

In the centre-of-mass frame these expressions simplify to

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{CM} = \frac{1}{2p_1^0 2p_2^0 |v_A - v_B|} \frac{\left|\vec{k}_1\right|}{(2\pi)^2 4E_{CM}} \overline{|\mathcal{M}(p_1, p_2 \to k_1, k_2)|^2}\,. \tag{D.21}$$

and

$$\sigma_{\text{total}} = \frac{1}{N_{\text{id}}!} \int \mathrm{d}\Omega \frac{1}{2p_1^0 2p_2^0 |v_A - v_B|} \frac{\left|\vec{k}_1\right|}{(2\pi)^2 4E_{CM}} \overline{|\mathcal{M}(p_1, p_2 \to k_1, k_2)|^2}\,. \tag{D.22}$$

Cross-Section of Gravitino Pair Production In the case of the reaction $\gamma\gamma \to \tilde{G}\tilde{G}$ we obtain

$$\sigma(\gamma\gamma \to \tilde{G}\tilde{G}) = \frac{1}{2} \int_0^\pi \mathrm{d}\theta\, \frac{k \sin\theta}{16\pi s^{3/2}} \overline{\left|\mathcal{M}(\gamma\gamma \to \tilde{G}\tilde{G})\right|^2} \quad \text{for} \quad m_{3/2} \neq 0\,, \tag{D.23}$$
$$\sigma(\gamma\gamma \to \tilde{G}\tilde{G}) = \frac{1}{2} \int_0^\pi \mathrm{d}\theta\, \frac{\sin\theta}{32\pi s} \overline{\left|\mathcal{M}(\gamma\gamma \to \tilde{G}\tilde{G})\right|^2} \quad \text{for} \quad m_{3/2} = 0\,, \tag{D.24}$$

where $k = \sqrt{\frac{s}{4} - m_{3/2}^2}$.

E

Formulae for Ideal Quantum Gases

For the computation of the gravitino luminosity in the SN core, we have to describe the initial state particles. The photons and even the neutrinos are in thermal equilibrium and can be described as an ideal quantum gas. In this section we state the relevant relations [85].

The average number of (identical) particles in a single-particle state i is given by the Fermi-Dirac or Bose-Einstein distribution function,

$$f_{\mathrm{f/b}}(\vec{p}) \equiv \begin{cases} \left(\exp\left[\frac{E_i-\mu}{T}\right]+1\right)^{-1}, & \text{(Fermi-Dirac)}, \\ \left(\exp\left[\frac{E_i-\mu}{T}\right]-1\right)^{-1}, & \text{(Bose-Einstein)}. \end{cases} \tag{E.1}$$

μ is denoting the chemical potential of the relevant particles.
With this functions we can express the number and energy density as

$$n_{\mathrm{f/b}} = \frac{g}{(2\pi)^3} \int \mathrm{d}^3 p f_{\mathrm{f/b}}(\vec{p}) \,, \tag{E.2}$$

$$= \frac{g}{2\pi^2} \int_m^\infty \frac{\sqrt{E^2 - m^2}}{\exp\left[\frac{E_i - \mu}{T}\right] \pm 1} E^2 \mathrm{d}E \,, \tag{E.3}$$

$$\rho_{\mathrm{f/b}} = \frac{g}{(2\pi)^3} \int \mathrm{d}^3 p f_{\mathrm{f/b}}(\vec{p}) E(\vec{p}) \,, \tag{E.4}$$

$$= \frac{g}{2\pi^2} \int_m^\infty \frac{\sqrt{E^2 - m^2}}{\exp\left[\frac{E_i - \mu}{T}\right] \pm 1} E \mathrm{d}E \,, \tag{E.5}$$

where we used $E^2 = |\vec{p}|^2 + m^2$.

Relativistic Limit For $T \gg m$ and $T \gg \mu$ we obtain

$$\rho = \begin{cases} \frac{\pi^2}{30} g T^4 \,, & \text{(Bose-Einstein)} \,, \\ \frac{7}{8} \frac{\pi^2}{30} g T^4 \,, & \text{(Fermi-Dirac)} \,, \end{cases} \tag{E.6}$$

$$n = \begin{cases} \frac{\zeta(3)}{\pi^2} g T^3 \,, & \text{(Bose-Einstein)} \,, \\ \frac{3}{4} \frac{\zeta(3)}{\pi^2} g T^3 \,, & \text{(Fermi-Dirac)} \,. \end{cases} \tag{E.7}$$

Here the Riemann zeta functions occurs. We just state that $\zeta(3) \approx 1.20206$.
This leads us to the average energy per particles,

$$\langle E \rangle \equiv \frac{\rho_{\mathrm{f/b}}}{n_{\mathrm{f/b}}} = \begin{cases} \frac{\pi^4}{30\zeta(3)} T \approx 2.7 T \,, & \text{(Bose-Einstein)} \,, \\ \frac{7\pi^4}{180\zeta(3)} T \approx 3.2 T \,, & \text{(Fermi-Dirac)} \,. \end{cases} \tag{E.8}$$

Non-relativistic Limit For $m \gg T$ we find no difference between bosons and fermions,

$$n = g \left(\frac{mT}{2\pi} \right)^{3/2} \exp\left[-(m - \mu)/T \right] \,, \tag{E.9}$$

$$\rho = m n \,. \tag{E.10}$$

F

Calculation of Cross-Sections with FeynCalc

We always use the Mathematica [70] package FeynCalc [69] for the squaring of long amplitudes and the calculation of cross-sections. Here we present the framework of such an Mathematica notebook as an example.

Calculation of Cross-Sections with FeynCalc

Preamble

Load FeynCalc:

```
$LoadFeynArts = False;
<< HighEnergyPhysics`FeynCalc`
```

Feynman Rules, Kinematics, Assumptions and necessary Substitution Rules

```
$Assumptions =
  κ > 0 && m_{3/2} > 0 && G > 0 && s > 0 && t > 0 && u > 0 && m_photino > 0 && M_p > 0 && c > 0 && d > 0;
GravitonPropagator[μ_, ν_, α_, β_, q_] :=
  I/(2 ScalarProduct[q, q]) (MT[μ, α] MT[ν, β] + MT[ν, α] MT[μ, β] - MT[μ, ν] MT[α, β]);
FermionPropagator[p_, m_] := (I (DiracSlash[p] + m))/(ScalarProduct[p, p] - m^2);
ScalarPropagator[p_, m_] := I/(ScalarProduct[p, p] - m^2);
PhotonPropagator[μ_, ν_, p_] := (- I MT[μ, ν])/(ScalarProduct[p, p]);
polarizationsums =
  {FCI[Pair[LorentzIndex[α_], Momentum[Polarization[x_, i]]] Pair[LorentzIndex[β_],
      Momentum[Polarization[x_, -i]]] :> PolarizationSum[α, β]]};
kinematicsET = {ScalarProduct[p1, p1] | ScalarProduct[p2, p2] | ScalarProduct[k1, k1] |
     ScalarProduct[k2, k2] → 0, ScalarProduct[p1, p2] | ScalarProduct[k1, k2] → s/2,
   ScalarProduct[p1, k1] | ScalarProduct[p2, k2] → s/4 (1 - Cos[ϕ]),
   ScalarProduct[p1, k2] | ScalarProduct[p2, k1] → s/4 (1 + Cos[ϕ])};
kinematics = {ScalarProduct[p1, p1] | ScalarProduct[p2, p2] → 0,
   ScalarProduct[k1, k1] | ScalarProduct[k2, k2] → m_{3/2}^2,
   ScalarProduct[p1, p2] → s/2, ScalarProduct[k1, k2] → s/2 - m_{3/2}^2,
   ScalarProduct[p1, k1] | ScalarProduct[p2, k2] → s/4 - Sqrt[s]/2 Sqrt[s/4 - m_{3/2}^2] Cos[ϕ],
   ScalarProduct[p1, k2] | ScalarProduct[p2, k1] → s/4 + Sqrt[s]/2 Sqrt[s/4 - m_{3/2}^2] Cos[ϕ]};
PolarizationTensorV[μ_, ν_, p_, m_] := - (GS[p] - m) . (MT[μ, ν] - (FV[p, μ] FV[p, ν])/m^2 -
    1/3 (DiracMatrix[μ] - FV[p, μ]/m^2 GS[p]) . (DiracMatrix[ν] - FV[p, ν]/m^2 GS[p]));
```

```
PolarizationTensorU[μ_, ν_, p_, m_] := - (GS[p] + m) . (MT[μ, ν] - FV[p, μ] FV[p, ν] / m^2 -
    1/3 (DiracMatrix[μ] - FV[p, μ]/m^2 GS[p]) . (DiracMatrix[ν] - FV[p, ν]/m^2 GS[p]));
replaceindices = {α → α2, β → β2, γ → γ2, δ → δ2, μ → μ2, ν → ν2, ρ → ρ2, σ → σ2,
   λ → λ2, ξ → ξ2, ω → ω2, i1 → i12, i2 → i22, i3 → i32, i4 → i42, j1 → j12, j2 → j22,
   l1 → l12, l2 → l22, m1 → m12, m2 → m22, n1 → n12, n2 → n22, o1 → o12, o2 → o22};
```

Amplitudes:

Write the amplitudes here:

```
M = Amplitude;
```

Cross-Section

Insert the amplitudes, the number of identical particles in the final state and the correct weight of the spin average and decide if you want to use the ET:

```
TimeS = SessionTime[];
IPFS = 2; (*Identical Particles in the Final State*)
M
(*(2/(3m_{3/2}^2) FV[k1,ν] FV[k2,μ]) M/.{POT→0}*)
% * ComplexConjugate[% /. replaceindices] // Expand;
"Averaging over the Photon Spins..."
1/4 %% //. polarizationsums // Contract;
Time1 = SessionTime[];
Row[{"Done (", Time1 - TimeS,
  "s). Insert a) Completion Relation and b) contract the Result:"}]
FermionSpinSum[%%% // Expand] (*//FullSimplify*);
(*%//Contract//FullSimplify;*)
Time2 = SessionTime[];
Row[{"Overall Time: (", Time2 - Time1, "s). Substitute the Polarization Tensor:"}]
%%% /. {(- POT + DiracGamma[Momentum[k1]]) -> PolarizationTensorV[ν, ν2, k1, m3/2],
   (POT + DiracGamma[Momentum[k2]]) -> PolarizationTensorU[μ2, μ, k2, m3/2]};
Row[{"Done. Calculating Traces..."}]
%% /. DiracTrace → Tr // Contract;
Time3 = SessionTime[];
Row[{"Done (", Time3 - Time2, "s). Substituting Kinematics..."}]
%%% /. kinematics // Simplify;
Time4 = SessionTime[];
Row[{"Done (", Time4 - Time3, "s). Integrating..."}]
```

```
Integrate[Sin[ϕ]/(16 π s^(3/2)) Sqrt[s/4 - m_{3/2}^2] %%%, {ϕ, 0, π}, GenerateConditions → False];
Time5 = SessionTime[];
Row[{"Done (", Time5 - Time4, "s). The result for the Cross Section:"}]
Simplify[CrossSection = 1/Factorial[IPFS] %%%, TimeConstraint → 600] // StandardForm
Row[{"Series Expansion for light gravitinos:"}]
FullSimplify[Series[%%, {m_{3/2}, 0, -4}] // Normal,
  Assumptions :→ s > 0 && m_{photino} > 0] // StandardForm
TimeE = SessionTime[];
Row[{"Overall Time: (", TimeE - TimeS, "s)."}]
NotebookSave[];
(*Quit[];*)
```

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Erklärung nach §18(8) der Prüfungsordnung für den Bachelor-Studiengang Physik und den Master-Studiengang Physik an der Universität Göttingen:

Hiermit erkläre ich, dass ich diese Abschlussarbeit selbstständig verfasst habe, keine anderen als die angegebenen Quellen und Hilfsmittel benutzt habe und alle Stellen, die wörtlich oder sinngemäß aus veröffentlichten Schriften entnommen wurden, als solche kenntlich gemacht habe.
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Göttingen, den 11.11.2013

(Timon Emken)