Infinite Tensor Products of $C_0(\mathbb{R})$: Towards a Group Algebra for $\mathbb{R}^{(\mathbb{N})}$.

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Abstract

The construction of an infinite tensor product of the C*-algebra $C_0(\mathbb{R})$ is not obvious, because it is nonunital, and it has no nonzero projection. Based on a choice of an approximate identity, we construct here an infinite tensor product of $C_0(\mathbb{R})$, denoted $\mathcal{L}_\mathcal{V}$, and use it to find (partial) group algebras for the full continuous representation theory of $\mathbb{R}^{(\mathbb{N})}$. We obtain an interpretation of the Bochner–Minlos theorem in $\mathbb{R}^{(\mathbb{N})}$ as the pure state space decomposition of the partial group algebras which generate $\mathcal{L}_\mathcal{V}$. We analyze the representation theory of $\mathcal{L}_\mathcal{V}$, and show that there is a bijection between a natural set of representations of $\mathcal{L}_\mathcal{V}$ and $\mathrm{Rep}\big(\mathbb{R}^{(\mathbb{N})}, \mathcal{H}\big)$, but that there is an extra part which essentially consists of the representation theory of a multiplicative semigroup $\mathcal{Q}$ which depends on the initial choice of approximate identity.

Keywords: C*-algebra, group algebra, infinite tensor product, topological group, Bochner–Minlos theorem, state space decomposition, continuous representation.