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Ensemble or Individual System, Collapse or no Collapse: A Description of a Single Radiating Atom

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Abstract

We use ordinary quantum mechanics to analyze a gedanken experiment of repeated photon measurements on an atom. The measurements are a short, but finite, time Δt apart. This leads to a coarse-grained time scale and to a description of photon counts from a single atom by a sample path of a classical stochastic process governed by quantum mechanics. It is shown that a collapse, or reduction, of the state vector at a no-photon ("null") measurement is not needed but may be used as a very convenient technical tool. We also show that within the coarse-grained time scale the axiomatic theory of continuous measurements of Davies and Srinivas can in the case of a radiating atom be obtained from ordinary quantum mechanics. Applications to macroscopic dark phases and quantum beats are indicated.
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1. Introduction

In recent years there have been exciting experiments on single atoms in Paul traps. Particularly interesting theoretical questions arise in connection with macroscopic dark periods or macroscopic quantum jumps of a single atom. Macroscopic dark periods of a single fluorescent atom were predicted by Dehmelt [1] for a system with two excited states, one rapidly decaying and the other metastable. Driving such a system by two lasers one intuitively expects frequent transitions from the ground state to the nonmetastable excited state with the subsequent emission of a spontaneous photon ("light period"). Once in a while there will be a transition to the metastable state, where the electron will stay for an extended period, and there will be no photons ("dark period", "electron shelving"). Quantum mechanically the situation is less clear than semiclassically because by the time development the atom will in general be in a coherent superposition of all three states, with an admixture of the rapidly decaying state always present, so that one may wonder if the dark periods really exist. These ideas have been analyzed semiclassically by the telegraph process [2] as well as quantum mechanically [3, 4, 5, 6]. Macroscopic dark periods were indeed found experimentally for single atoms in a Paul trap [7], confirming a spectacular quantum effect.

In the following we use only ordinary quantum mechanics. By this we mean the statistical interpretation as well as the reduction of the state vector after a measurement [8]. According to the statistical interpretation, as understood here, one deals with probability statements which are experimentally verified as frequencies, or relative numbers of events, in an ensemble. Instead of considering many systems one may also measure on an individual system, then prepare it again in the same way, measure again, and so on. In the case of interest here this would mean an ensemble of many atoms each with its own radiation field and laser, or a single atom with radiation field and laser, observed infinitely long, then prepared again as before, observed again, and so on.

For reductions we take the von Neumann-Lüders rule [9]. For example, if one measures the energy of a system in the state $|\psi\rangle$ and finds a particular eigenvalue of the energy which is degenerate, then the state immediately after the measurement is given by the projection of the state $|\psi\rangle$ onto the eigenspace of the respective eigenvalue, with ensuing normalization. Such a reduction is surely an idealization of the measurement process and only a substitute for a detailed theory of the measuring apparatus.

Such a change of the state vector will, of course, influence the results of subsequent measurements. As an example we consider a spin $\frac{1}{2}$ in a magnetic field $B = (B, 0, 0)$ in x-direction. The Hamiltonian is given by

$$H == 1/2\,\hbar B \cdot \sigma = 1/2\,\hbar B \sigma_1$$

As initial state $|\varphi_0\rangle$ we take the one with spin up, $\sigma_3\,|\varphi_0\rangle = |\varphi_0\rangle$. Then, at time t,

$$|\,\varphi_t\rangle = \exp\{-i\sigma_1 B_1 t/2\}\;|\varphi_0\rangle\;.$$

We now imagine the 3-component of the spin measured in two different ways.

a) We measure at time $T = \pi/B$ only. This gives

$$\langle\ \sigma_3/2\ \rangle_T = -1/2$$

b) We measure first at time $T/2$, then perform a reduction of the wave function, and measure again at time T. This gives

$$\langle\ \sigma_3/2\ \rangle_T = 0$$

which differs radically from the previous outcome.

We now consider an atom which radiates photons. One then has the problem that if the photons are detected by a counter or seen by the eye of an observer one performs, in principle, a measurement, with all the consequences of the theory. Moreover, due to the stochastic nature of the emission times, how does one know *when* to measure?

2. Null Measurements

We imagine a general N-level atom possibly illuminated by one or several lasers and use an ideal detector of efficiency 1 to measure the photons emitted. We start at $t = 0$ and assume the first photon to be detected at time t_1. This is obviously a photon measurement, and one may try to describe it quantum mechanically as follows. At time $t_0 = 0$ one has a no-photon state $|0_{ph}\rangle$ together with an atomic state $|\psi_A\rangle$. Until time t_1 one uses the unitary time development,

$$U(t_1, t_0) \ |0_{ph}\rangle |\psi_A\rangle,$$

and then one would have a reduction at time t_1. The probability $P_0(t)$ to have found no photon at time t, $t \leq t_1$, is then

$$P_0(t) = \sum_{j_A} |\langle j_A | \langle 0_{ph} | U(t, t_0) | 0_{ph}\rangle | \psi_A\rangle|^2$$

where the sum is over all atomic states. Defining the projector $\mathbb{P}_0$ by

$$\begin{aligned} \mathbb{P}_0 &\equiv \sum_{j_A} |0_{ph}\rangle |j_A\rangle\langle j_A| \langle 0_{ph}| \\ &\equiv |0_{ph}\rangle \, 1_A \, \langle 0_{ph}| \end{aligned} \tag{1}$$

one can write $P_0(t)$ as

$$\boxed{P_0(t) = \| \mathbb{P}_0 \, U(t, t_0) \, |0_{ph}\rangle \, |\psi_A\rangle \|^2} \tag{2}$$

The right-hand side of this has been calculated by Porrati and Putterman [6]. The probability $P_0(t)$ is important for the determination of photon rates and for dark periods, as first pointed out in Ref. [3].

There is an objection to this. In order to *know* that there had been no photon before t_1, one would have had to open the detector between 0 and t_1 without detecting a photon. Opening the detector and *not* finding a photon, however, is also a measurement which may be called a "null measurement" [10]. Should each of these null measurements not also be accompanied by a reduction? In the above procedure leading to (2) they are not manifestly taken into account.

How many of these null measurements does one need? Ideally, infinitely many, and ultimately this line of reasoning would lead to "continuous measurements" [11]. However, it is well-known that the von Neumann-Lüders rule leads to difficulties with continuous measurements since in the idealized limit of measurements repeated infinitely fast it leads to a freezing of the state, the so-called quantum Zenon effect [12].

By an axiomatic extension of quantum mechanics Davies and Srinivas [13] have constructed a theory of continuous measurement which is adapted to counting rates. But for any particular situation one needs a phenomenological input or some intuition to obtain the explicit form. We will come back to this theory in the Section 5.

3. From Ensemble to Single System

We return to the N-level system of the last section and consider a *gedanken* experiment. To avoid freezing of the state due to the quantum Zenon effect we open the detector at instances a very short – but finite – time Δt apart and perform, at each null measurement, an explicit reduction of the state vector. To obtain limits on Δt we require that Δt should be

(i) much shorter than the life time of a level, i.e., much less than $10^{-8}s$;

(ii) large compared to the time it takes a photon to travel the distance of an atomic diameter (essentially the correlation time in quantum optics).

We thus arrive at

$$\Delta t \cong 10^{-16} - 10^{-12}\ s\ .$$

Therefore we have up to 10^8 additional reductions per second on top of the 10^8 photons or so.

Reductions and sample paths. We assume that the single atom from above, with its radiation field, is a member of an ensemble $\mathcal{E}$ described by the initial state $|0_{ph}\rangle\ |\psi_A\rangle$. The atoms may or may not be driven by external pumping. At times Δt apart we imagine a measurement on each system of $\mathcal{E}$. By $\mathcal{E}_0^{(\Delta t)}$ we denote the subensemble of all systems for which *no* photon was found at time Δt. Similarly, $\mathcal{E}_0^{(n\Delta t)}$ denotes the subensemble of systems for which at times Δt, $2\Delta t, \cdots$, $n\Delta t$ no photon was found. Clearly one has $\mathcal{E}_0^{(\Delta t)} \supset \cdots\cdots \supset \mathcal{E}_0^{(n\Delta t)}$. According to the von Neumann-Lüders rule the subensemble $\mathcal{E}_0^{(\Delta t)}$ is described by the state vector

$$I\!P_0\ U(\Delta t, 0)\ \ |0_{ph}\rangle |\psi_A\rangle\ /\ \|\cdot\|$$

where the projector $I\!P_0$ is given by (1). The n-th subensemble $\mathcal{E}_0^{(n\Delta t)}$ is described by

$$I\!P_0\ U(n\Delta t, (n-1)\Delta t)\ I\!P_0\ \cdots\ I\!P_0\ U(\Delta t, 0)\ \ |0_{ph}\rangle |\psi_A\rangle\ /\ \|\cdot\| \tag{3}$$

which shows the intermittent unitary time development interrupted by repeated reductions in between. The first subensemble $\mathcal{E}_0^{(\Delta t)}$ has relative magnitude

$$\|\ I\!P_0\ U(\Delta t, 0)\ |0_{ph}\rangle |\psi_A\rangle\|^2$$

which in the statistical interpretation is the probability of finding no photon at time Δt. The relative magnitude of subensemble $\mathcal{E}_0^{(n\Delta t)}$ compared to $\mathcal{E}$ is

$$\boxed{\begin{aligned} P_0(n\Delta t)\ \equiv\ \|\ I\!P_0\ U(n\Delta t, (n-1)\Delta t)\ I\!P_0\ U((n-1)\Delta t, (n-2)\Delta t)\cdots \\ \cdots I\!P_0\ U(2\Delta t, \Delta t) I\!P_0\ U(\Delta t, 0)\ |0_{ph}\rangle\ |\psi_A\rangle\ \|^2 \end{aligned}} \tag{4}$$

which gives the probability of finding no photon at the times $\Delta t, \cdots$, $n\Delta t$. This expression is quite different from the previous one in (2).

At each measurement on the individual system under consideration chance decides according to the probabilities $P_0(\Delta t)$, $\cdots$, $P_0(n\Delta t)$, $\cdots$ whether or not a photon is detected. This behavior can be simulated by flipping a coin weighted with the conditional probabilities for each measurement. Once a photon is detected and absorbed our individual system becomes a member of a new ensemble $\mathcal{E}'$. How to describe the new ensemble is in general quite a subtle question and will be discussed elsewhere. Here we will assume for simplicity that the system is reset to $|0_{ph}\rangle\,|0_A\rangle$ where $|0_A\rangle$ is the atomic ground state [14]. With no external pumping the state remains constant after t_1. If a driven atom starts out from the ground state and if at time $t_1 = (n+1)\Delta t$, say, the first photon is detected then the procedure starts again, with a time t_2 for the next photon detection, and so forth.

Thus in the above approach the photon and no-photon detection times for a single atom form a sample path of a classical stochastic process which is governed by quantum mechanics. Without pumping the sample path terminates. In the simple case of a driven atom which is reset to the ground state after a photon detection one obtains a path of a renewal process since after each photon detection the memory is lost. In general, however, this need not be so.

Evaluation of $P_0(n\Delta t)$. We can rewrite the state vector of the n-th subensemble in (3) as

$$|0_{ph}\rangle\,\langle 0_{ph}\,|U(n\Delta t,(n-1)\Delta t)\,|0_{ph}\rangle \quad\cdots\quad \langle 0_{ph}\,|U(\Delta t,0)\,|0_{ph}\rangle \quad |\psi_A\rangle \;/\; \|\cdot\|$$

$$\equiv\; |0_{ph}\rangle\,|\psi_A(n\Delta t)\rangle \;/\; \|\cdot\| \tag{5}$$

where the expressions $\langle 0_{ph}\,|\,U(n\Delta t,(n-1)\Delta t)\,|0_{ph}\rangle$ are now purely atomic operators and where $|\psi_A(n\Delta t)\rangle$ is a vector in the atomic space of norm less than 1, due to the repeated reductions. One has

$$P_0(n\Delta t) \;=\; \|\;\psi_A(n\Delta t)\;\|^2\;. \tag{6}$$

Since Δt is small one can use ordinary perturbation theory to evaluate (5). A standard Hamiltonian is [15]

$$H \;=\; H_A \;+\; H_F \;+\; D\cdot(E \;+\; E_L(t)) \tag{7}$$

where H_A is the purely atomic part, H_F the radiation field part, E the quantized field, $E_L(t)$ a possible classical field of lasers, and D the atomic dipole operator,

$$D \;=\; e\sum\; |i_A\rangle\langle i_A|\,X\,|j_A\rangle\langle j_A|\;\;.$$

Going over to the interaction picture with respect $H_0 \;=\; H_A \;+\; H_F$ one obtains in second order

$$\langle 0_{ph}\,|U(m\Delta t,(m-1)\Delta t)\;|0_{ph}\rangle$$

$$=\; e^{-iH_A m\Delta t}\Big\{1_A \;-\; i\hbar^{-1}\int_{(m-1)\Delta t}^{m\Delta t} dt'\;\langle 0_{ph}\;|\;H_I(t')\;|0_{ph}\rangle$$

$$-\;\hbar^{-2}\int_{(m-1)\Delta t}^{m\Delta t} dt' \int_{(m-1)\Delta t}^{t'} dt''\;\langle 0_{ph}\,|H_I(t')H_I(t'')\;|0_{ph}\rangle\Big\}\;\; e^{iH_A(m-1)\Delta t}$$

The expression in the curly brackets is easy to evaluate as indicated in footnote [16] where for $\Delta t \to 0$ the quantum Zenon effect appears automatically; a particular case is evaluated in [17]. E.g., for a three-level V-system with states $|0\rangle$, $|1\rangle$, $|2\rangle$ and with transition frequencies ω_1, ω_2 and laser frequencies ω_{L1}, ω_{L2} the curly bracket can be written as [18]

$$1_A - i\hbar^{-1}\int_{(m-1)\Delta t}^{m\Delta t} dt' \Big[\sum_j \Big(1/2\Omega_j \ |j\rangle\langle 0| \ e^{i(\omega_j-\omega_{L_j})t'} + h.c. - \frac{1}{2} i\, A_j\, |j\rangle\langle j|\Big) - i\,\gamma_{12}\ |1\rangle\langle 2|\ e^{-i\,\omega_{21}t'} - i\,\gamma_{21}\,|2\rangle\langle 1|\ e^{i\omega_{21}t'}\Big]$$

$$\cong \exp\Big\{-i\,\hbar^{-1}\int_{(m-1)\Delta t}^{m\Delta t} dt' \,\big[\cdots\big]\Big\}$$

where

$$\gamma_{ij} = \langle 0 \mid X \mid i\rangle \cdot \langle j \mid X \mid 0\rangle\ \omega_j^3\ e^2/6\pi\epsilon_0\hbar c^3 \tag{8}$$

and where $A_j = 2\gamma_{jj}$ are the Einstein coefficients and Ω_j the Rabi frequencies, which are proportional to the laser amplitudes. Now the product over m from 1 to n can be performed, leading to a time-ordered expression of the form

$$|\psi_A(n\Delta t)\rangle = \mathcal{T}\exp\Big\{-i\,\hbar^{-1}\int_0^{n\Delta t} dt'\ H_{red}(t')\Big\}\,|\psi_A(0)\rangle \tag{9}$$

The "reduced Hamiltonian" H_{red} is nonhermitian. In particular, for the V-system one obtains for H_{red} in matrix form with respect to the basis $|0\rangle$, $|1\rangle$, $|2\rangle$

$$\hbar\ H_{red} = \begin{pmatrix} 0 & \Omega_1/2e^{i\omega_{L1}t} & \Omega_2/2\ e^{i\omega_{L2}t} \\ \Omega_1/2\ e^{-i\omega_{L1}t} & E_1 - i\ A_1/2 & -i\ \gamma_{12} \\ \Omega_2/2\ e^{-i\ \omega_{L2}t} & -i\ \gamma_{21} & E_2 - i\ A_2/2 \end{pmatrix} \tag{10}$$

We now introduce a "coarse-grained" time,

$$t = n\Delta t\ ,\quad n = 0,1,2,\cdots\ . \tag{11}$$

Then (9) can be written as

$$\frac{d}{dt}\ |\psi_A(t)\rangle = -i\,\hbar^{-1}\ H_{red}\ |\psi_A(t)\rangle \tag{12}$$

and one has

$$P_0(t) = \||\psi_A(t)\rangle\|^2\ . \tag{13}$$

Collapse or no collapse: How does this result compare with that calculated from (2)? Surprisingly, within the *coarse-grained* time scale our $P_0(t)$, obtained by n-fold reductions, coincides completely with that obtained with no reductions [19]. The n-fold repeated reductions do not seem to have an effect except for the temporal coarse-graining! The approach of this section, however, has the advantage of mathematical simplicity since straightforward perturbation theory can be used.

Ensemble or individual system: The coarse-grained time can, for all questions relating to time differences much larger than Δt, be considered as practically continuous. On the coarse-grained time scale the detection of photons from an atom can thus be described by a sample path of a classical stochastic process with continuous time, a process which is governed by quantum mechanics. Without external pumping these paths terminate and then it is clear that one can make no definite statements about an individual system. With external pumping, however, this is possible due to the ergodic property of the process. Ergodicity allows one to replace time averages over a sample path by ensemble averages which in turn can be calculated by probability theory. For a renewal process this is more or less evident, and for the general case ergodicity is physically expected. This explains that although in the statistical interpretation quantum mechanics deals with ensembles it can make certain definite predictions for a *single* driven atom.

4. Applications

Macroscopic dark periods [3, 4, 6, 17]. For the Dehmelt V-system the off-diagonal elements γ_{ij} in H_{red} can be neglected. This is most easily seen by going to an interaction picture with respect to an auxiliary Hamiltonian $H_0' = \omega_{L1} \,|1\rangle\langle 1\,| + \omega_{L2}\,|2\rangle\langle 2\,|$ which removes the time dependence from the Ω_i-terms and adds an $\exp\{\pm i(\omega_{L1} - \omega_{L2})t\}$ to the γ_{ij}-terms. This produces rapid oscillations which lead to negligible contributions. With their neglect one obtains a $P_0(t)$ identical to that of Cohen-Tannoudji and Dalibard [3], and their analysis applies. Due to the fact that one level is metastable, level 2 say, one has $A_2 \ll A_1$ and this leads $P_0(t)$ to split into the sum of two parts, one rapidly decaying roughly like $\exp\{-A_1 t\}$, the other very slowly decaying, roughly like $\exp\{-A_2 t\}$, and with a very small factor in front. There is thus a small probability to reach this region where the second term prevails — i.e. very many photon detections will be needed —, but once this region is reached one has to wait a very long time for the next photon since its probability density is $w_1(t) = -P_0'(t)$. During this dark period, $|\,\psi_A(t)\rangle$ is not completely in the metastable state $|\,2\rangle$ but has a $|\,1\rangle$-component. Hence in contradiction to the semi-classical electron-shelving picture there is a finite probability – in fact it can be approximately 1/2 – that the next photon does not originate from the transition metastable to ground state [6, 17].

Quantum beats. We consider a three-level V-system whose upper levels have only a very small energy difference $\hbar\delta\omega$ and no laser ($\Omega_i = 0$). We consider the decay from one of the excited states. In this case the off-diagonal terms γ_{ij} in (10) become important. $P_0(t)$ will now contain oscillating terms which leads to a non-exponential decay, the well-known quantum beats. Here it turns out that these beats also occur for the decays of levels 1 and 2 separately, not only for coherent superpositions as required in some textbooks [20].

Macroscopic dark periods without metastable state [21]. Again we consider a V-system with very small upper-level separation $\hbar\delta\omega$ and irradiate it with a *single* laser tuned to the vicinity of the upper levels. The Rabi frequency is denoted by Ω. We now assume in addition that the transition dipole moments are *parallel*. For $\delta\omega \ll \Omega$ light and dark periods are predicted. Their mean duration T_L and T_D can be explicitly calculated for arbitrary laser detuning. In particular, if the transition dipole moments are equal and if the laser is tuned to the $0-1$ or $0-2$ transition one finds

$$\begin{aligned} T_L &= 4\,\Omega^2/A_1(\delta\omega)^2 \\ T_D/T_L &= \Omega^2/2(\delta\omega)^2 \,. \end{aligned}$$

If the laser is tuned exactly halfway between the upper levels the surprising phenomenon is predicted, for any $\delta\omega$, that after the emission of a number of photons the atom will stop fluorescing completely ($T_D \to \infty$). This is related to a nonabsorption resonance in gases [22, 23].

5. Connection with the Continuous Measurement Theory of Davies and Srinivas

Davies and Srinivas [13] have extended the axiomatics of quantum mechanics by postulates for 'homogeneous quantum counting processes'. In particular, their postulates imply the existence of two 'superoperators' J and S_t which map trace class operators to trace class operators and satisfy certain properties. For an individual system of an ensemble described by a density matrix ρ their meaning is as follows. $Tr(S_t\rho)$ is the probability of finding no counting event in $[0,t]$, and the probability density $w(t_1,\cdots,t_n;[0,t])$ for finding a counting event exactly at the times $t_1,\cdots,t_n$ in $[0,t]$ is given by

$$w(t_1,\cdots,t_n;[0,t]) \;=\; Tr\left(S_{t-t_n}\;J\;S_{t_n-t_{n-1}}J\;\cdots\;J\;S_{t_2-t_1}\;J\;S_{t_1}\rho\right) \tag{14}$$

For a particular system J and S_t have to be determined phenomenologically or by intuition.

Since we have an explicit expression for $P_0(t)$ we can *derive* the form of the superoperators J and S_t. Only ordinary quantum mechanics is used and no additional postulates are required. We illustrate this for the three-level V-system, two excited states coupled to a common ground state, with two lasers. Let the initial atomic state be ρ. The corresponding $P_0(t;\rho)$, valid until the detection of the first photon, is obtained from (9) and (10) by carrying these equations over to density matrices in an obvious way. Then, after the detection of the i-th photon the atom is each time reset to the ground state, and the corresponding $P_0(t-t_i;|0\rangle)$ is obtained from (10) with $|\psi_A\rangle = |0\rangle$. By standard arguments of classical probability theory one then finds for the n-photon probability density $w(t_1,\cdots,t_n;\;[0,\;t])$ [17]

$$w(t_1,\cdots,t_n;[0,\;t]) \;=\; P_0(t-t_n;|0\rangle)\,w_1(t_n-t_{n-1};|0\rangle)\;\cdots\;w_1(t_2-t_1;|0\rangle)\,w_1(t_1;\rho) \tag{15}$$

where $w_1 = -P'_0$.

We now define superoperators J and S_t by

$$J\,\rho \;:=\; |0\rangle\langle 0 \mid Tr\{\, i(H_{red} - H^*_{red})\rho \,\} \tag{16}$$

$$S_t\,\rho \;:=\; \mathcal{T}\exp\left\{-i\hbar^{-1}\int_0^t dt'\; H_{red}(t')\right\}\;\rho\;\left[\mathcal{T}\exp\left\{-i\hbar^{-1}\int_0^t dt'\; H_{red}(t')\right\}\right]^* \tag{17}$$

We note that J is time independent, by (10). Now, first of all it is apparent that $Tr\,(S_t\,\rho)$ coincides with $P_0(t;\rho)$. Inserting (16) and (17) on the right-hand side of (14) one finds by a calculation similar to one in Ref. [17] that this agrees with (15).

Using only ordinary quantum mechanics we have in this way exhibited operators that satisfy the requirements of the quantum counting process of Ref. [13]. There is, however, a severe conceptual *proviso*. In our approach we are dealing with a coarse-grained time, and the above seemingly continuous variables $t_1, \cdots, t_n$ in (15) are, truly speaking, discrete. We thus arrive at the conclusion that the "continuous" measurement theory of Ref. [13] can, at least in the case considered here, be derived from ordinary quantum mechanics if one relaxes the "continuous" and goes over to a coarse-grained time scale.

6. Discussion

Our intuitive idea is that it should make no difference for the photon statistics whether or not all photons are actually observed once they are sufficiently far away from the atom and do no longer interact with it. In a cavity with reflecting walls this would evidently not be true. Therefore we think that the results of Section 3, together with ordinary probability theory, can also be applied to situations where only a part of the photons are actually detected. This is substantiated by the result that for the photon statistics it makes no difference whether or not reductions are performed at no-photon ("null") measurements. With these reductions, however, elementary perturbations theory can be used since Δt is very small, and this simplifies the analysis considerably. In this sense the null reductions may be considered as technical tool.

No attempt has been made to give a detailed theory of the measurements, but at each measurement and depending on its outcome a straightforward reduction of the state vector according to the von Neumann-Lüders rule is carried out. These reductions put an individual system each time into a particular subensemble, and this branching into subensembles may depend on the system under consideration. In the simple case considered in Section 3 the atom is reset to the ground state after a photon detection, and for a driven atom one then obtains a sample path of a renewal process . In general, however, the resetting will not always be to the same state and might in fact even be time dependent. In this way we arrive at the result that on a time scale much coarser than Δt the photon emissions of an atom can be regarded as a sample path of a classical stochastic process obtained from ordinary quantum mechanics. Ergodicity allows one to replace time averages over a sample path by ensemble averages, and such quantities can thus be calculated for a *single* radiating atom.

It should be pointed out, however, that our gedanken experiment with its repeated reductions and temporal coarse-graining and its reduced description of the atomic state is not applicable to all questions encountered for a single radiating atom. The term 'emission' of a photon appears to be imprecise and should be replaced by 'detection' since it is doubtful whether it makes sense quantum mechanically to speak about emission without an actual observation. If one is interested in spectral distributions of the emitted light repeated observations would cause changes, as is clear from the time-energy uncertainty relation. For such observables like the spectrum, which in a sense is complementary to photon counting, one will need other facets of the complete wave function of atom plus radiation field. The true wave function contains all information and gives a 'holistic' description of all aspects of the system while some partial aspects, as the photon statistics, may be amenable to a simplified description.

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[11] Indeed, such a view seems to be implicit in Refs. [5, 6].

[12] B. Misra and E.C.G. Sudarshan, J. Math. Phys. **18**, 756 (1977); W.M. Itano, D.J. Heinzen, U.J. Bollinger, and D.J. Wineland, Phys. Rev. A **41**, 2295 (1990); T. Petrosky, S. Tasaki, and I. Prigogine, Physica A **170**, 306 (1991); E. Block and P.R. Berman, Phys. Rev. A **44**, 1466 (1991); V. Frerichs and A. Schenzle, Phys. Rev. A **44**, 1962 (1991).

[13] E.B. Davies, Commun. Math. Phys. **15**, 277 (1969); **22**, 51 (1971); M.D. Srinivas and E.B. Davies, Opt. Acta **28**, 981 (1981); **29**, 235 (1982).

[14] This will be true for an atomic V-system which consists of two excited states optically coupled to a common ground state, as in the Dehmelt system.

[15] R. Loudon, *The Quantum Theory of Light*. 2. Edition, Clarendon Press, Oxford 1983.

[16] The first integral contains the contribution of the laser fields only. The double integral consists of terms

$$\int_{(m-1)\Delta t}^{\Delta t} dt' \int_{(m-1)\Delta t}^{t'} dt'' \sum_{k\lambda} g_{ijk\lambda}\, g_{lmk\lambda}\, \exp\{-i(\omega_k - \omega_{ij})t'\} \exp\{i(\omega_k - \omega_{lm})t''\}\, |i\rangle\langle j\,|m\rangle\langle l|$$

where $\hbar\, \omega_{ij} = E_i - E_j$ and where the coupling constants $g_{ikk\lambda}$ contain the transition dipole moments for the i- and j-levels. The exponentials can be written as

$$\exp\{i(\omega_{ij} - \omega_{lm})t'\}\, \exp\{-i(\omega_k - \omega_{lm})(t' - t'')\} \ .$$

The integral over $\tau = t' - t''$ gives approximately $\pi\, \delta(\omega_k - \omega_{\ell m})$, up to a line shift which can be neglected or incorporated in the levels. Alternatively one can use that

$$\sum_{k\lambda} g_{ijk\lambda}\, g_{mlk\lambda}\, \exp\{-i(\omega_k - \omega_{lm})\tau\}$$

is sharply peaked at $\tau = 0$ and essentially vanishes if τ is much larger than the correlation time. This gives the lower limit for Δt required in (ii). The relevance of the correlation time was pointed out to us by R. Reibold (private communication). If, however, Δt is much smaller than the correlation time, in particular if $\Delta t \to 0$, then the double integral behaves as $(\Delta t)^2$ and does not contribute. Only the purely atomic part remains and there is no photon emission, in accordance with the quantum Zenon effect.

[17] T. Wilser, Dissertation, Göttingen 1991.

[18] Some rapidly oscillating terms have been neglected. This corresponds to the rotating-wave approximation.

[19] With hindsight this is maybe not so surprising since without reductions one also obtains a semi-group equation. Ref. [6] uses as interaction $P \cdot A$ instead of the dipole form $D \cdot E$ in (7). In Eq. (8) for γ_{ij} one then has to replace ω_j^3 by $\omega_i \omega_j^2$. But this is of no consequence since for small level separation the expressions agree and for larger level separation the γ_{ij}-terms can be neglected, as explained in Section 4.

[20] Cf., e.g., P. Meystre and M. Sargent III, *Elements of Quantum Optics.* Springer 1990.

[21] G.C. Hegerfeldt and M.B. Plenio, preprint, Göttingen 1991.

[22] G. Orriols, Nuovo Cimento B**53**, 1 (1979); E. Arimondo and G. Orriols, Lett. Nuovo Cimento **17**, 333 (1976).

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