EUROPHYSICS LETTERS 7 July 1998

Europhys. Lett., **29** (6), pp. 1-∞ (1995)

Reply to comment on "Evidence for nontrivial ground-state structure of 3d $\pm J$ spin glasses"

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(received ; accepted)

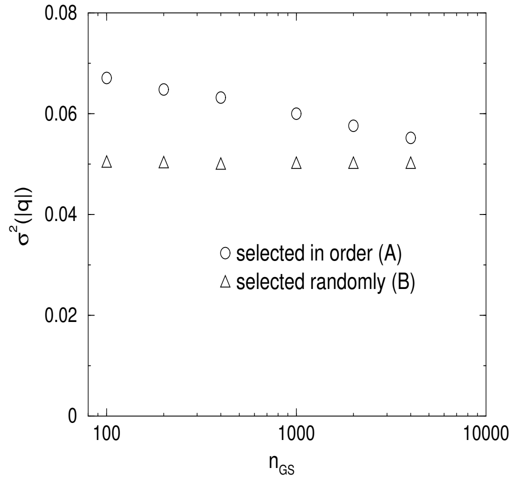
PACS. 75.10.Nr – Spin glasses and other random models.
PACS. 75.40.Mg– Numerical simulation studies .
PACS. 02.10.Jf – General mathematical systems.

In a recent letter [1] the ground-state landscape of three-dimensional $\pm J$ Ising spin glasses was investigated using the genetic Cluster-exact Approximation (CEA) algorithm. In the foregoing comment A.W. Sandvik argued that the method does not ensure, that all ground states are reached with the same probability. To clarify this point we calculated a huge number of ground states for small systems of size $N = L^3 = 3^3, 4^3, 5^3$. 1000 realizations of the disorder for $L = 3, 4$ and 100 realizations for $L = 5$ were considered. For each realization all ground states were calculated by performing up to 10^5 runs of the genetic CEA algorithm. Each particular state was stored only once and it was counted in how many runs it occurred. We obtained that different states have significant different probabilities of being found, as claimed by A.W. Sandvik.

Before performing further analysis a systematic local search was applied to add possibly missing ground states which are related to already existing states by flips of free spins. Finally a $L = 3$ realization exhibits 25 different ground states on average. For a $L = 4$ realization on average 240 states were found and 6900 states for $L = 5$.

To investigate, how the statistical weights of the ground states affect physical quantities, we evaluated the average overlap distribution $P(|q|)$ in two ways. For both cases at most n_{GS} ground states were included in the calculation for each realization. Two different methods of selecting the states were applied: method A selects the first n_{GS} states in the order of their appearance in the calculation. That means that ground states with a higher probability of being found are included in the calculation more frequently since they appear on average before the less probable states. For the second method n_{GS} states were selected independently and randomly among all ground states for each realization. Thus, method B garanties that all ground states enter the result with the same weight. Fig. 1 shows the variance $\sigma^2(|q|)$ of $P(|q|)$. Method B (lower points) gives always the same result, while method A shows a systematic dependence of $\sigma^2(|q|)$ on n_{GS}. Hence, the states which occur with higher probability give rise to a larger variance than the other states. The comparison with the result of method B ensures, that this dependence is not merely an effect caused by the number of states included in the calculation.

The correct result of the variance for $L = 3, 4, 5$, where all ground states enter with the same



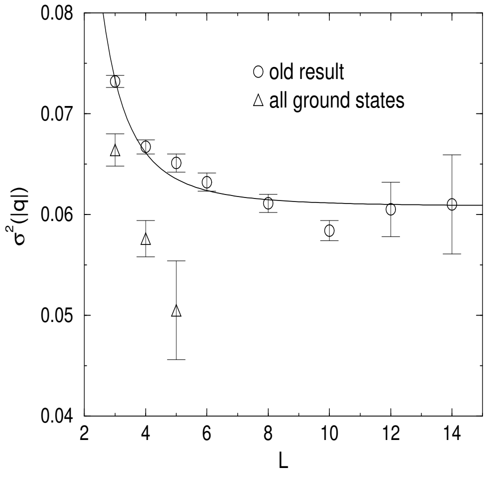


Fig. 1. – Average Variance of $P(|q|)$ as function of number n_{GS} of ground states used for each realization in the calculation ($L = 5$, 100 realizations). For the upper points (at most) the first n_{GS} different states in the order of their appearance in the calculation were used. For the lower points n_{GS} states were selected randomly and independently per realization from all ground states.

Fig. 2. – Variance of $P(|q|)$ as function of system size L. The upper points were obtained using pure genetic CEA. The lower points were calculated by selecting 100 states from all ground states for each realization.

weight, is compared to the old data [1] in Fig. 2. The correct values are located below the old data points. Thus, it is questionable whether the width of the distribution of overlaps remains finite for the infinite system. Since only results for small systems are available at the moment it is not possible to draw any conclusions about the true finite size behavior of $\sigma^2(|q|)$. This applies also to multi-canonical simulations, because the statistics of ground states if not known for this method.

It is possible to extend the genetic CEA by generating additional ground states locally [2] by applying a T=0 Monte Carlo simulation, so that the correct thermodynamic behavior is obtained. Using this combination of methods it should be possible to investigate the true thermodynamic behavior at $T = 0$ of $P(|q|)$ for larger sizes.

Acknowledgements. – The author thanks A.W. Sandvik for his inspiring comment and K. Battacharya for interesting discussions. He was supported by the Graduiertenkolleg "Modellierung und Wissenschaftliches Rechnen in Mathematik und Naturwissenschaften" at the *Interdisziplinäres Zentrum für Wissenschaftliches Rechnen* in Heidelberg and the *Paderborn Center for Parallel Computing* by the allocation of computer time.

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